Massachusetts Institute of Technology
Department of Mechanical Engineering
2.12 Introduction to Robotics

Problem Set No.8

Out: November 21, 2005 Due: November 30, 2005

Problem 1

Shown below is a vehicle similar to the 2.12 mobile robot having a pair of powered wheels and a frictionless caster. The radius of the wheels is *r=3 cm*, while the distance between the two wheels is 2*b=20 cm*. The angular velocity of the right wheel is ω_r , and that of the left wheel is ω_l . Each powered wheel is equipped with a shaft encoder to measure the angular velocity. Answer the following questions.

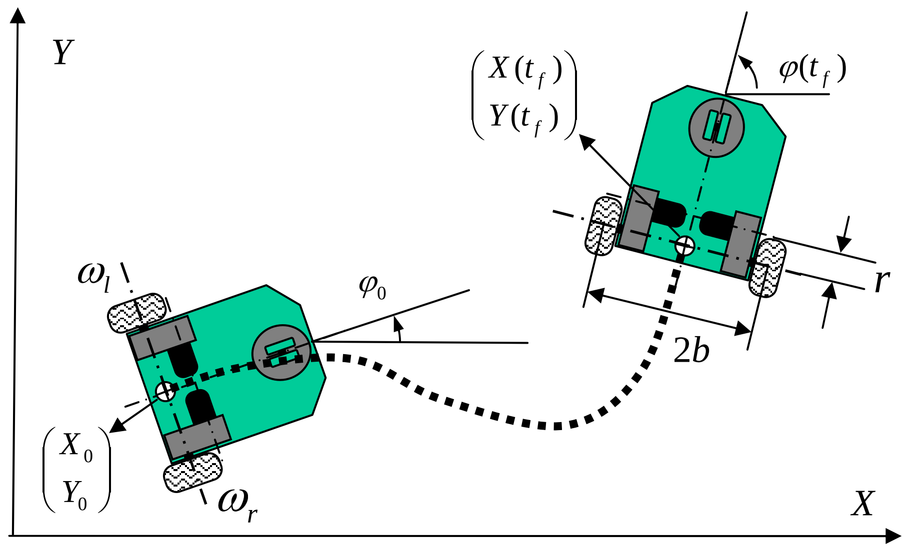

Figure 1 Vehicle trajectory

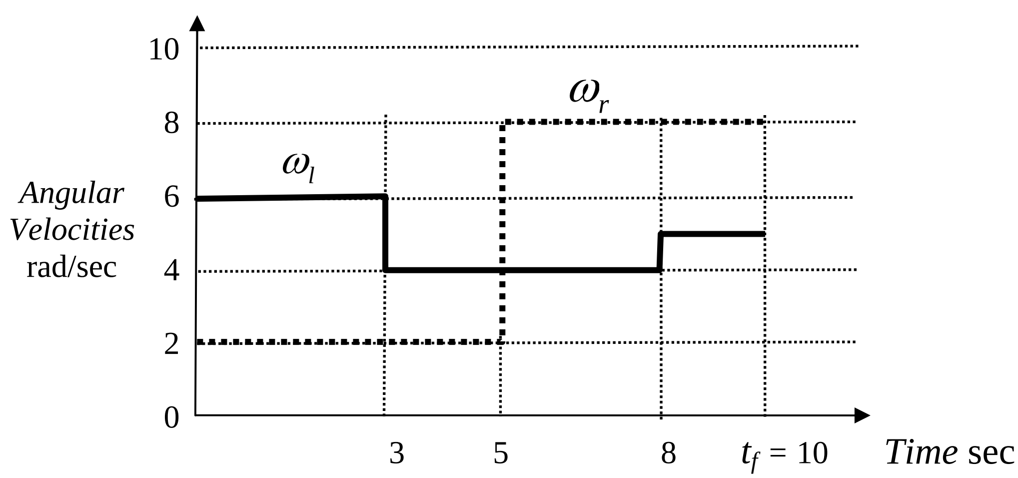

Figure 2 Time profiles of the wheel velocities

a). At time t = 0, the vehicle was at position $X_0 = 20\,cm, Y_0 = 20\,cm$ with reference to the inertial reference frame *O-XY* and at orientation $\varphi_0 = 0$ measured from the positive X axis. See Figure 1. Then the vehicle moved. The time profiles of the wheel angular velocities during the movement were recorded, as shown in Figure 2. Compute the position and orientation of the vehicle at time $t_f = 10$ sec based on the time profiles shown in Figure 2. Assume no slip.

To go back to the initial position and orientation, X_0, Y_0, φ_0, a feedback control law is now employed. Let us consider the following control method.

As illustrated in Figure 3, let α be the angle between the direction of the vehicle, i.e. line *AB*, and the direction of the destination from the current position of the vehicle, line *AC*.

$$\alpha = \arctan 2\left[Y_0 - Y(t),\ X_0 - X(t)\right] - \varphi(t)$$

The primary goal is to reduce the distance between the current position $X(t), Y(t)$ and the destination X_0, Y_0,

$$D = \sqrt{(X(t) - X_0)^2 + (Y(t) - Y_0)^2}$$

To reduce this distance D the vehicle should move in the direction given by angle α. At the same time the vehicle should be oriented in the direction of $\varphi_0 = 0$ at the destination. Therefore, the vehicle should reduce the difference in orientation:

$$\beta = \varphi_0 - \varphi(t)$$

during the movement towards the destination.

To combine all these, let us consider the following feedback law:

$$v = k_D D$$

$$\dot{\varphi} = k_\alpha \alpha + k_\beta \beta$$

where v is the vehicle forward velocity and $\dot{\varphi}$ is the angular velocity of the vehicle rotation. Answer the following questions.

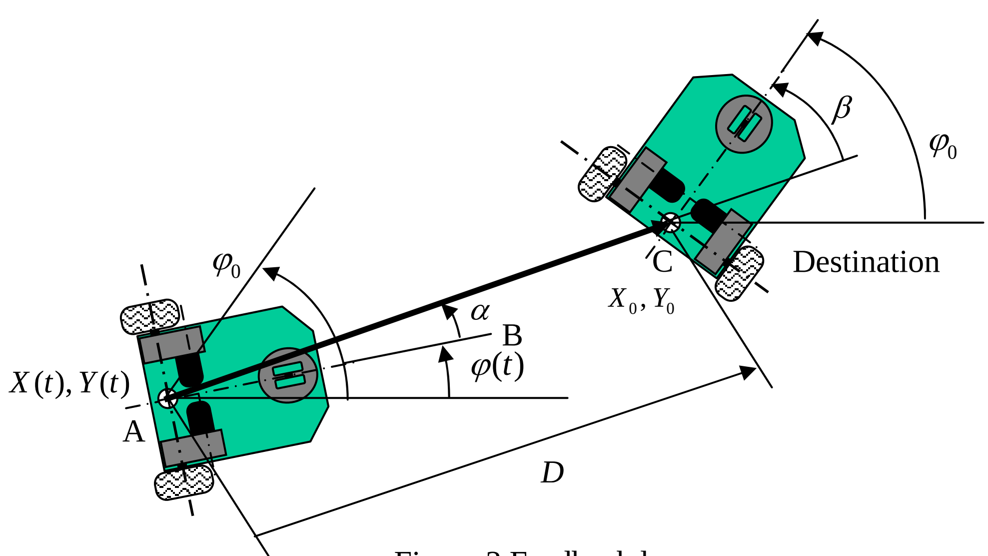

Figure 3 Feedback law

b). Obtain the Jacobian relating the vehicle forward velocity *v* and rotation velocity $\dot{\varphi}$ to the angular velocities of the right and left wheels, ω_r and ω_l.

c). Sketch an approximate trajectory of the vehicle from the final position obtained in Part a), i.e. $X(t_f), Y(t_f), \varphi(t_f)$, back to the original position and orientation, X_0, Y_0, φ_0. Find appropriate values for the feedback gains, k_D, k_α, k_β.

For Extra Credit:

d). Discuss whether the vehicle can reach the exact destination when the feedback gains are changed. What will happen if $k_\alpha < k_D$?

e). If $|\alpha| < \pi/2$, moving forward may be better than moving backward. What will happen if the vehicle moves backward when $\pi/2 < |\alpha| < \pi$? Considering these alternative routes, how do you modify the control law in order to move quickly towards the destination?

Problem 2

The objective of this assignment is to build the dynamic model of the 2.12 arm being used for the final project, and obtain feedforward torques for manipulating an end-effecter in a vertical plane. As you already know, both actuators of the 2.12 arm are fixed to the base link, and the actuator torque of the second motor, τ_2, is transmitted from joint 1 to joint 2 through a belt-pulley mechanism. Actuator displacements ϕ_1 and ϕ_2, which are absolute angles measured from the base axis, are used as generalized coordinates, and actuator torques τ_1 and τ_2 correspond to the actuator displacements, forming virtual work: $\delta Work = \tau_1 \cdot \delta\phi_1 + \tau_2 \cdot \delta\phi_2$.

a) Obtain mass properties of each link, i.e. m_i, ℓ_{ci}, I_i, as defined in Figure 4. Figure 6 illustrates the disjointed arm links (Details are ignored for computing the mass properties). Each arm link consists of an aluminum bar of 275 mm x 50 mm x 20 mm and two masses at both ends of the link. For simplicity, the masses at both ends are treated as mass particles having no moment of inertia. Obtain the mass, the location of the center of mass, and the moment of inertia about the center of mass, m_i, ℓ_{ci}, I_i, for each link.

b) Obtain feedforward actuator torques for compensating for the gravity load of the arm when displacements ϕ_1 and ϕ_2 are measured.

For extra credit:

c) Obtain equations of motion in terms of generalized coordinates ϕ_1 and ϕ_2 and actuator torques τ_1 and τ_2. Discuss why no Coriolis term is involved in the equations of motion.

d) Consider the cosine curves shown in Figure 7 for the trajectories of ϕ_1 and ϕ_2. Compute the angular velocities and accelerations, $\dot{\phi}_1$, $\dot{\phi}_2$ and $\ddot{\phi}_1$, $\ddot{\phi}_2$, along the trajectories, and then obtain the feedforward torques for tracking the trajectories from time $t = 0$ to $t = t_f$.

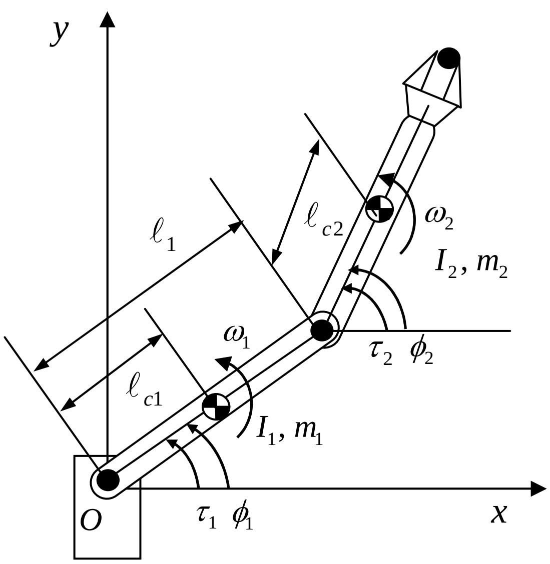

Figure 4 Variables and parameters of the 2.12 arm

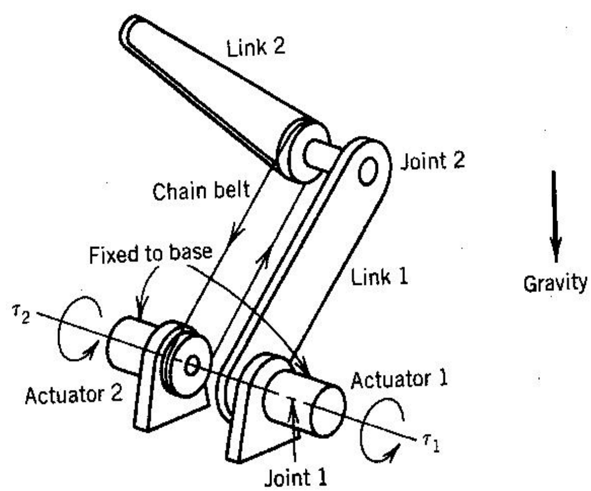

Figure 5 Mechanism of the arm

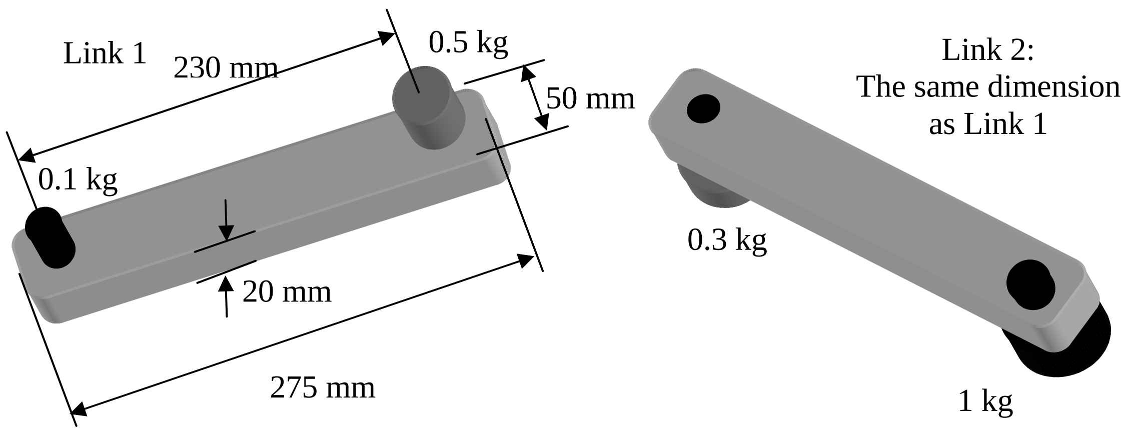

Figure 6 Link dimensions and masses attached to the links

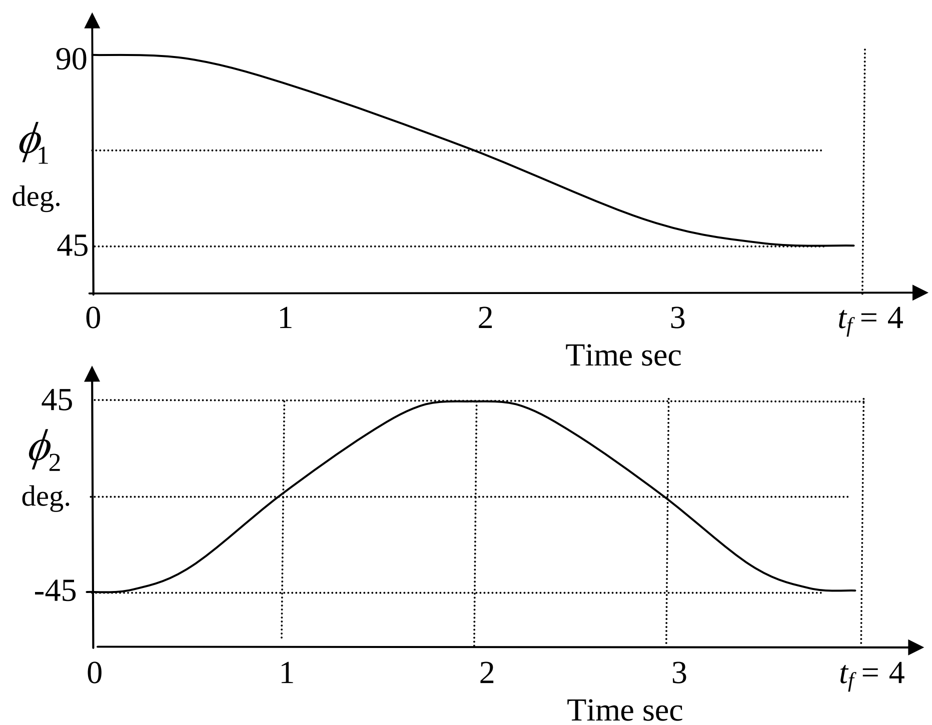

Figure 7 Trajectories