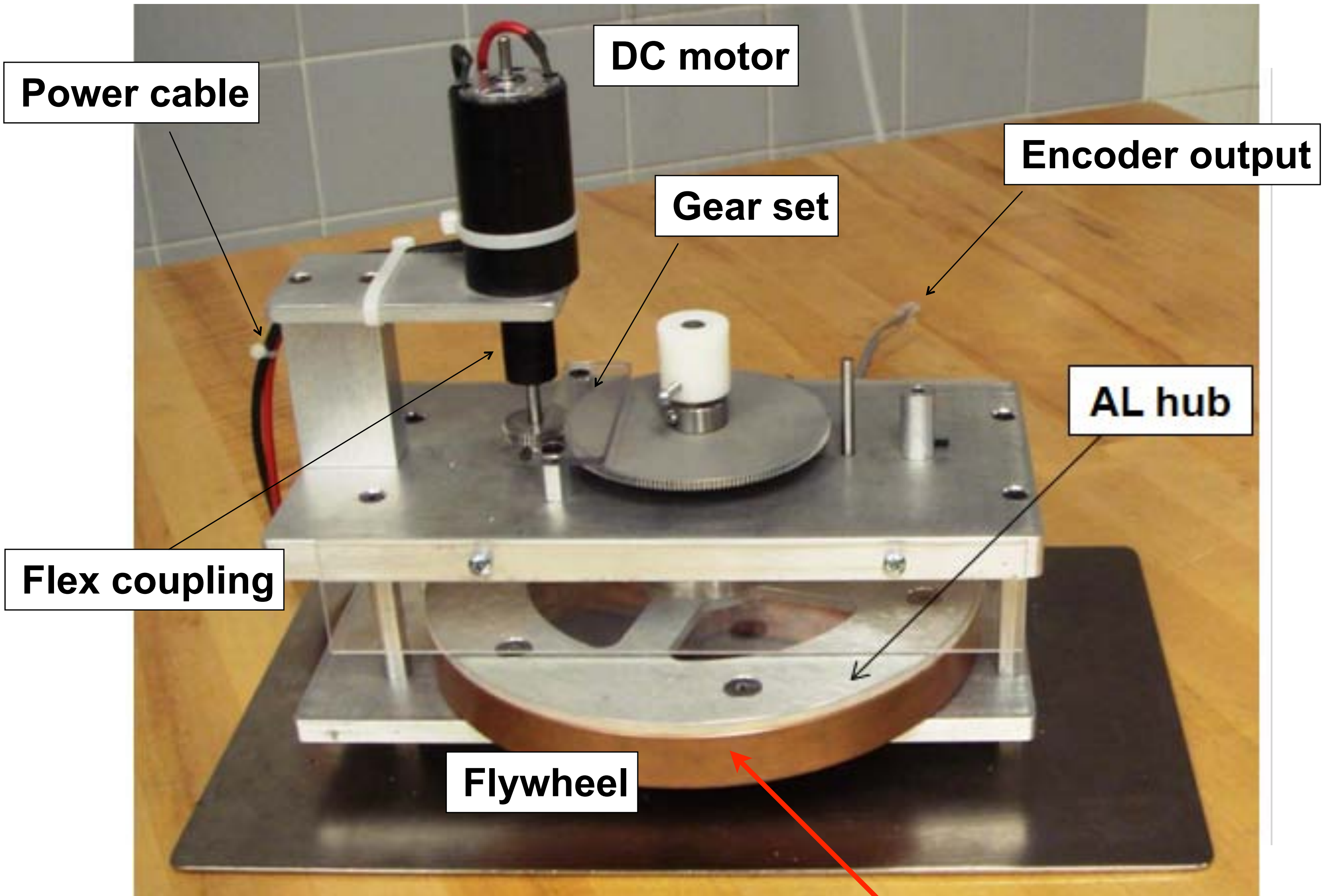
Rotational Plant

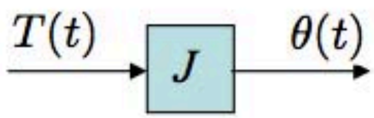
Read the Description of the Experimental Rotational Plant

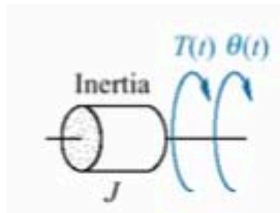


$$\boxed{J \approx 3 \times 10^{-2}\ \text{kg m}^2}$$

Mechanical system components: rotation

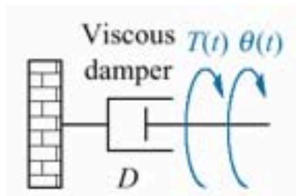
- Inertia





- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE (Newton's law): $m\dfrac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} \equiv m\ddot{\theta}(t) = T(t)$

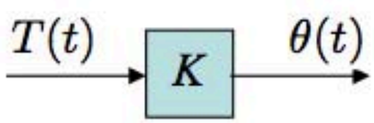
- Damper (rot.)
  - viscous

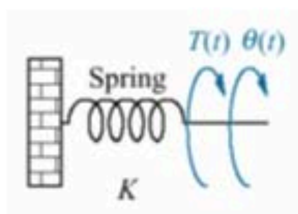


- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE: $D\dot{\theta}(t) = T(t)$

  - Coulomb → Component ODE: $D_c \mathrm{sgn}[\dot{\theta}(t)] = T(t)$
  - drag → Component ODE: $D_d \left|\dot{\theta}(t)\right| \dot{\theta}(t) = T(t)$

- Spring (rot.)



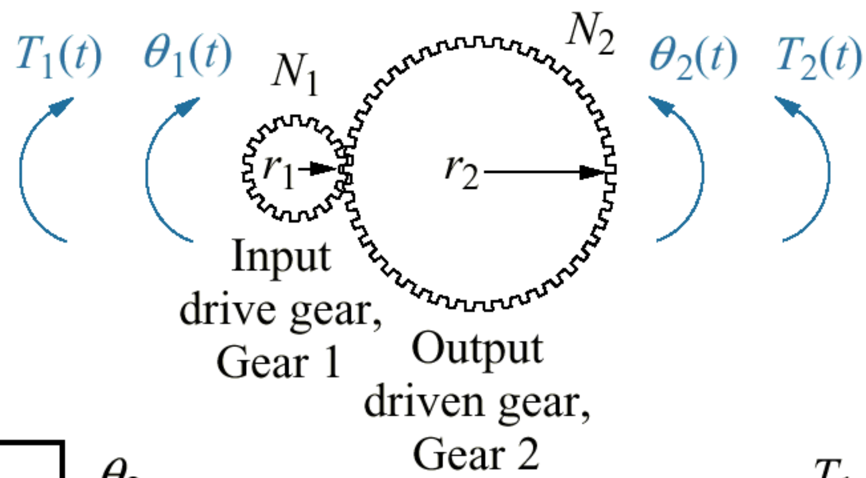


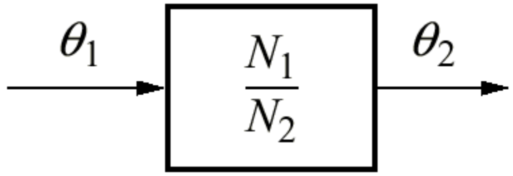
- Component input: **torque** $T(t)$
- Component output: **angle** $\theta(t)$
- Component ODE (Hooke's law): $K\theta(t) = T(t)$

- Gear (next page)

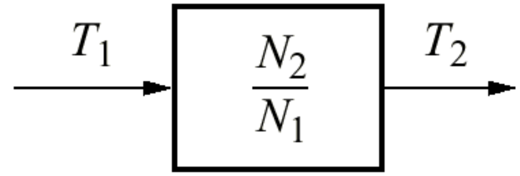
Nise Table 2.5

Mechanical system components: rotation: gears





(*a*)



(*b*)

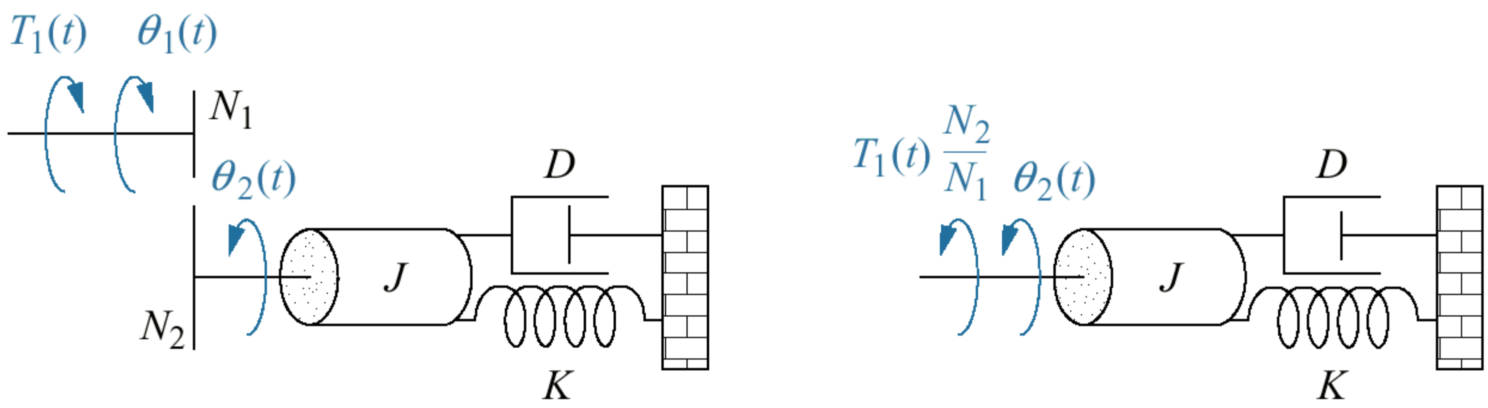
- Component input: **angle** $\theta_1(t)$
- Component output: **angle** $\theta_2(t)$
- Component ODE: $$\theta_2 = \frac{N_1}{N_2}\theta_1$$

- Component input: **torque** $T_1(t)$
- Component output: **torque** $T_2(t)$
- Component ODE: $$T_2 = \frac{N_2}{N_1}T_1$$

Question: Why is $T_1\theta_1 = T_2\theta_2$?

Nise Figure 2.27, 2.28

Gear transformations



(a) **(b)**

Let T_2 denote the torque applied to the left of the inertia J. The equation of motion is

$$J\ddot{\theta}_2 + D\dot{\theta}_2 + K\theta_2 = T_2,$$

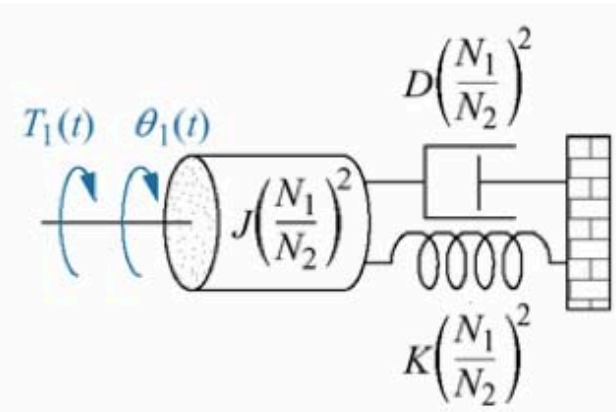
while from the gear equations we have

$$T_2 = T_1\frac{N_2}{N_1} \qquad \text{and} \qquad \theta_2 = \theta_1\frac{N_1}{N_2}.$$

Combining, we obtain

$$\left[\left(\frac{N_1}{N_2}\right)^2 J\right]\ddot{\theta}_1 + \left[\left(\frac{N_1}{N_2}\right)^2 D\right]\dot{\theta}_1 + \left[\left(\frac{N_1}{N_2}\right)^2 K\right]\theta_1 = T_1.$$

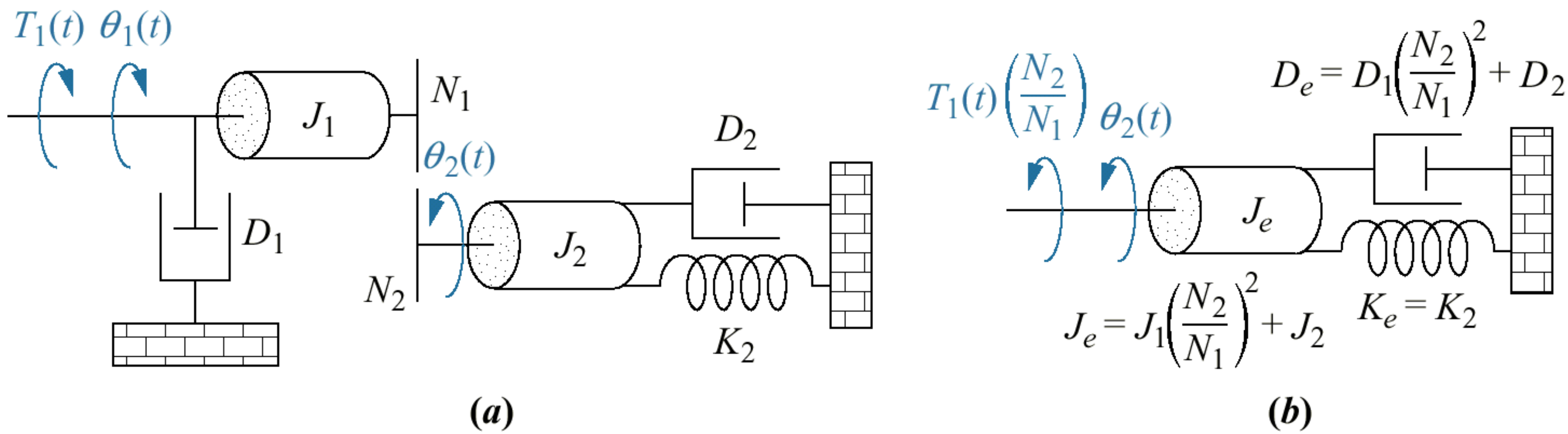
This is the equation of motion of the equivalent system shown in (c).



(c)

Nise Figure 2.29

Rotational mechanical system: example



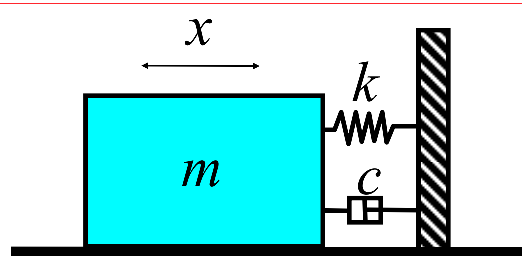
Equation of motion:

$$\left[\left(\frac{N_1}{N_2}\right)^2 J_1 + J_2\right]\ddot{\theta}_2 + \left[\left(\frac{N_1}{N_2}\right)^2 D_1 + D_2\right]\dot{\theta}_2 + K_2\theta_2 = \left(\frac{N_2}{N_1}\right)T_1.$$

Nise Figure 2.30a-b

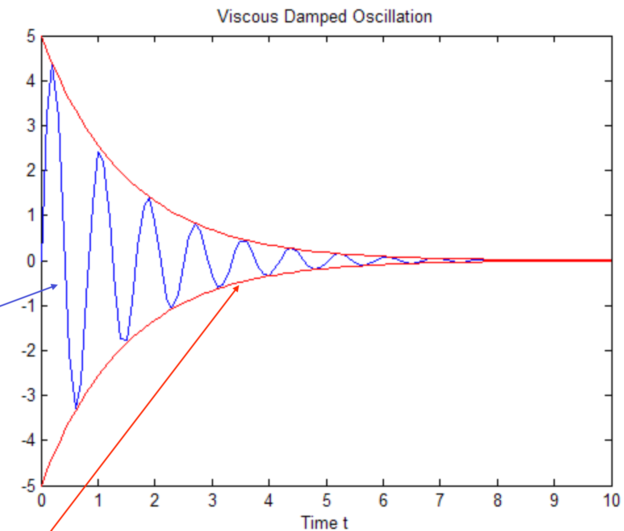
Damping Model

•Viscous Damping



$$m\ddot{x} + c\dot{x} + kx = F(t)$$

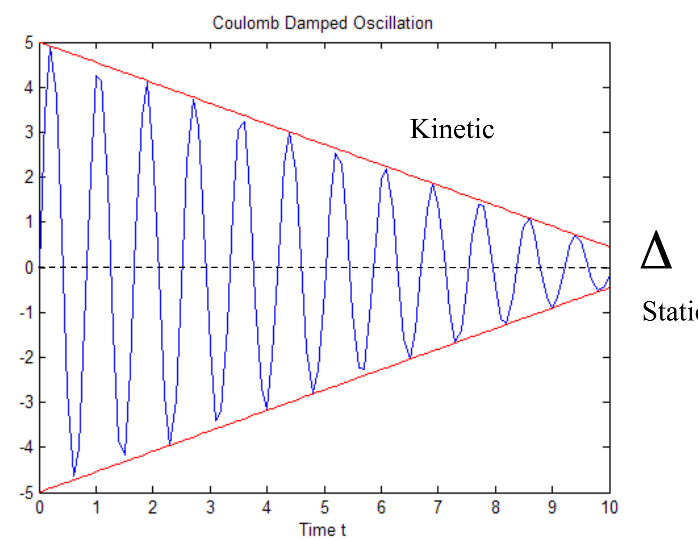
$$x = e^{-(c/2m)t}\left(Ae^{\sqrt{(c/2m)^2-(k/m)t}} + Be^{-\sqrt{(c/2m)^2-(k/m)t}} \right)$$



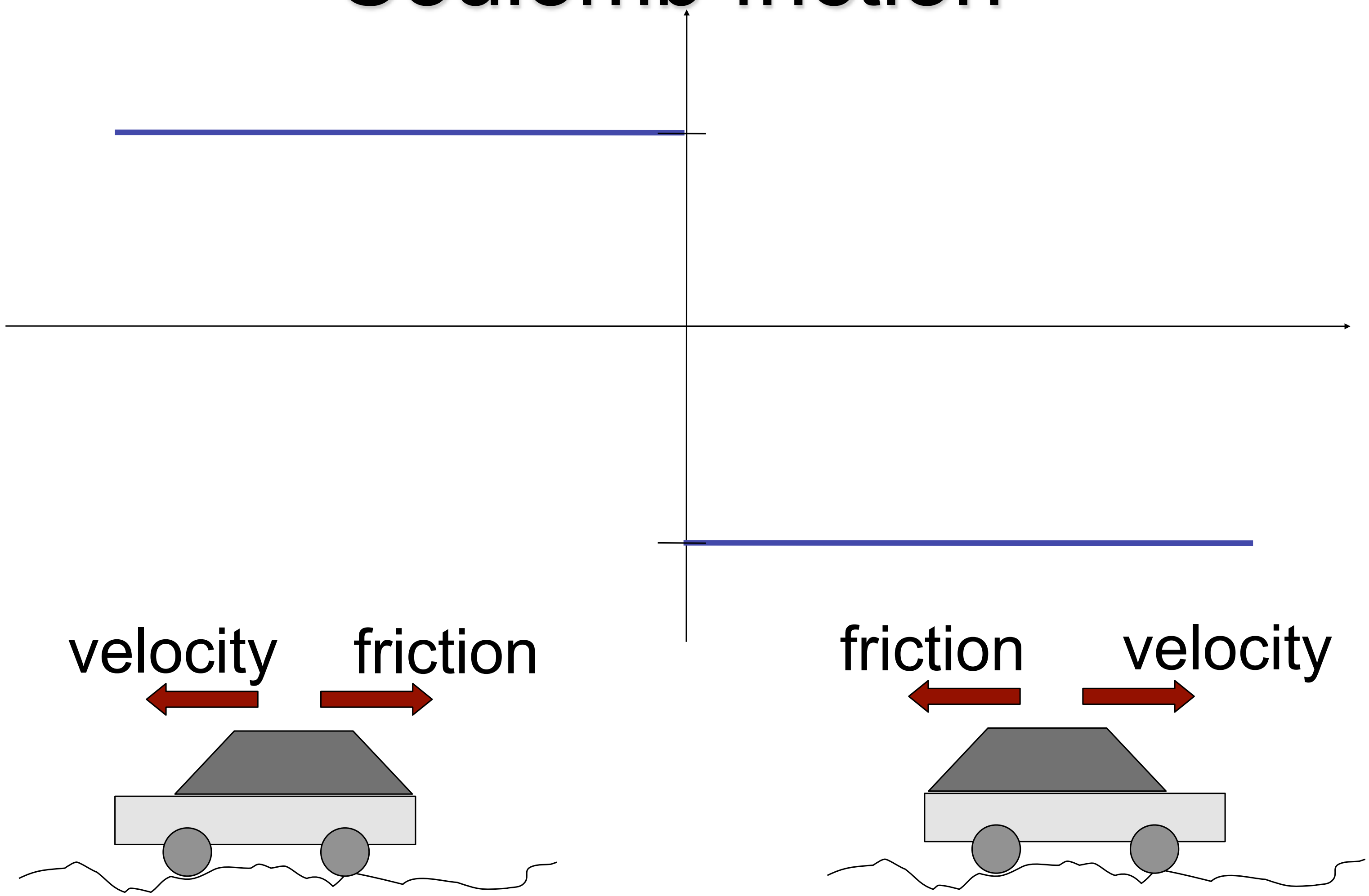
•Coulomb Damping

…Coulomb damping results from the **sliding** of two dry surfaces. The friction generated by the relative motion of the two surfaces is a source of energy dissipation. It is opposite to the direction of motion and is <u>independent of surface area, displacement or position, and velocity</u>…

$$F_c = \mu N$$



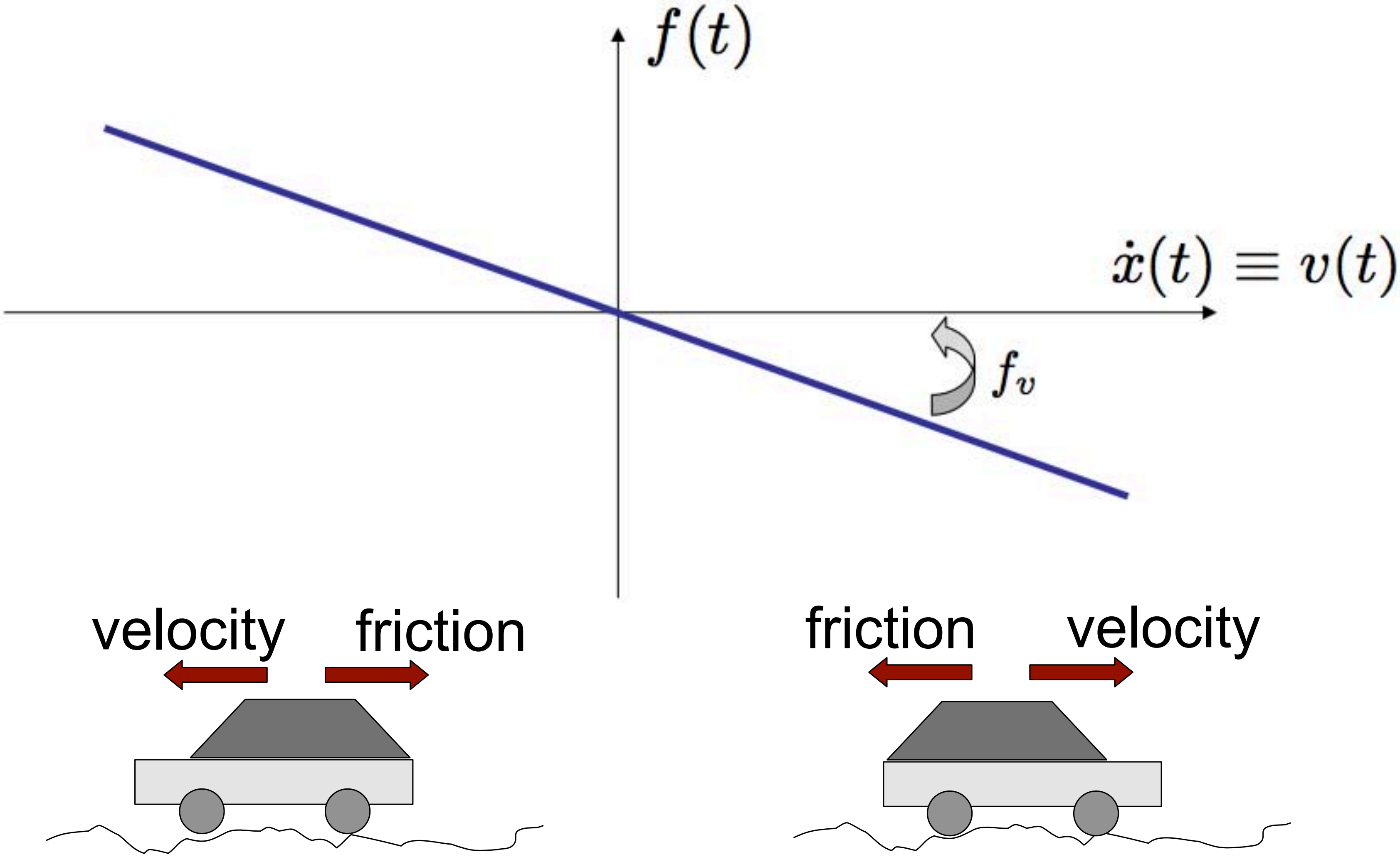
Coulomb friction



Coulomb friction is in opposite direction to the velocity; the magnitude of the friction force is <u>independent</u> of the magnitude of the velocity

Example: [Block sliding on a rough surface]

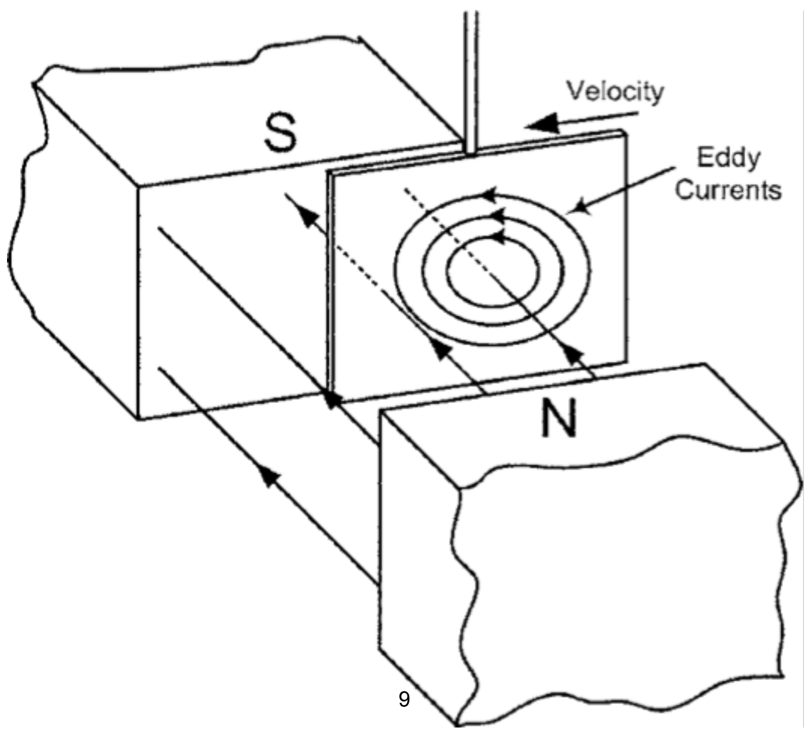
Viscous friction



Viscous friction is in opposite direction to the velocity; the magnitude of the friction force is <u>proportional</u> to the magnitude of the velocity

Eddy Currents ➡ Viscous friction

- Eddy currents are generated when there is relative motion between a conducting object and a magnetic field. The rotating currents in the conducting object are due to electrons experiencing a Lorentz force that is perpendicular to their motion and the magnetic field ($\mathbf{F}=q\mathbf{v}\times\mathbf{B}$).
- The Lorentz force results in current in the radial direction on the flywheel; these currents, since the wheel is turning, result in an opposing magnetic field and a force resisting the motion.
- The Eddy current and the resisting force are both proportional to the velocity $|\mathbf{v}|$; therefore, they resist motion in a way that is exactly equivalent to viscous friction.



In-class experiment

- Familiarize with the laboratory equipment and software tools

- Study the frictional characteristics of the motor, gear train, and bearings in the flywheel system

- Explore the effect of damping on the flywheel system

Optical Encoders

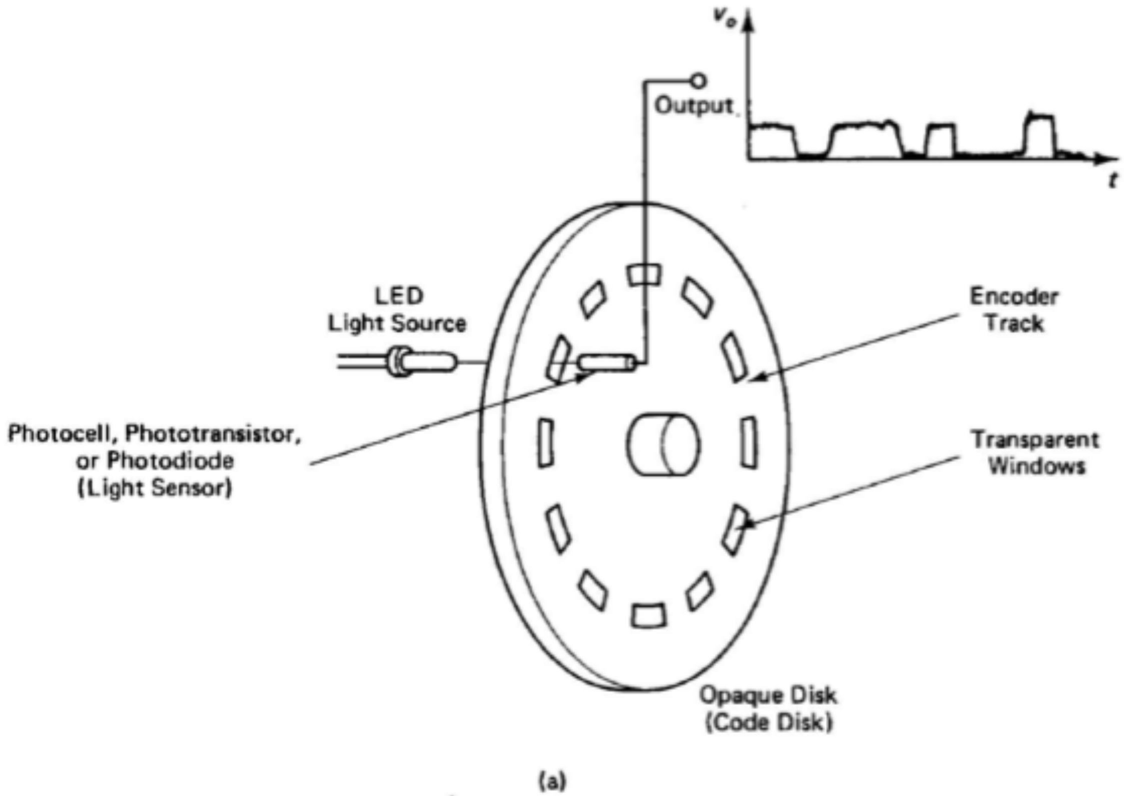
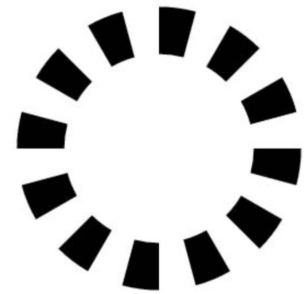
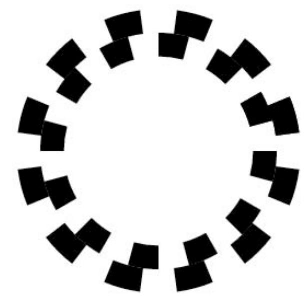


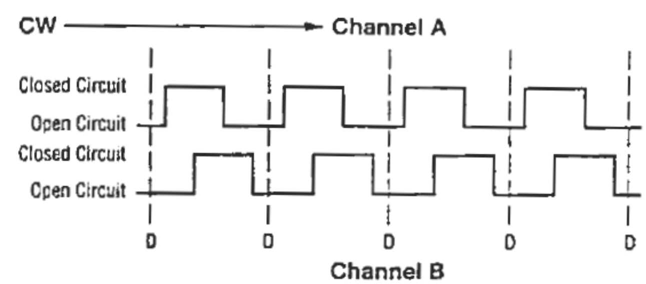
Figure 5.1. (a) Schematic representation of an optical encoder. (b) Components of an incremental optical encoder (courtesy of Hewlett-Packard Conmpany).



Regular phase



Quadrature phase



US Digital Optical Encoder E6S-2048-187

- Quick, simple assembly and disassembly
- Rugged screw-together housing
- Positive finger-latching connector
- Accepts .010" axial shaft play
- Tracks from 0 to 100,000 cycles/sec
- 64 - 2500 CPR | 256 to 10,000 PPR
- 2 channel quadrature TTL squarewave outputs
- Optional index (3rd channel)
- -40 to +100C operating temperature
- Fits shaft diameters from 2mm to 1"

Max speed = 2,400 rpm

Converting Tachometer reading

MITMECHE

You will monitor the angular velocity profile using the ETach2 electronic tachometer that is attached to the rotary encoder on the flywheel shaft.

It produces an analog voltage v_o proportional to the shaft speed $\omega \rightarrow V_o = K_t\omega$

where the tachometer constant $K_t = 0.016$ volts/rpm

$$\boxed{\omega(t) = \frac{V_0(t)}{K_t}}$$

Procedure

- Spin the flywheel by hand, and record the angular velocity decay $\omega(t)$, using the computer-based Chart Recorder(VI),(Remember to convert the Chart Recorder output to angular velocity.)

- Repeat the same procedure with one and two magnets (damper) on the flywheel

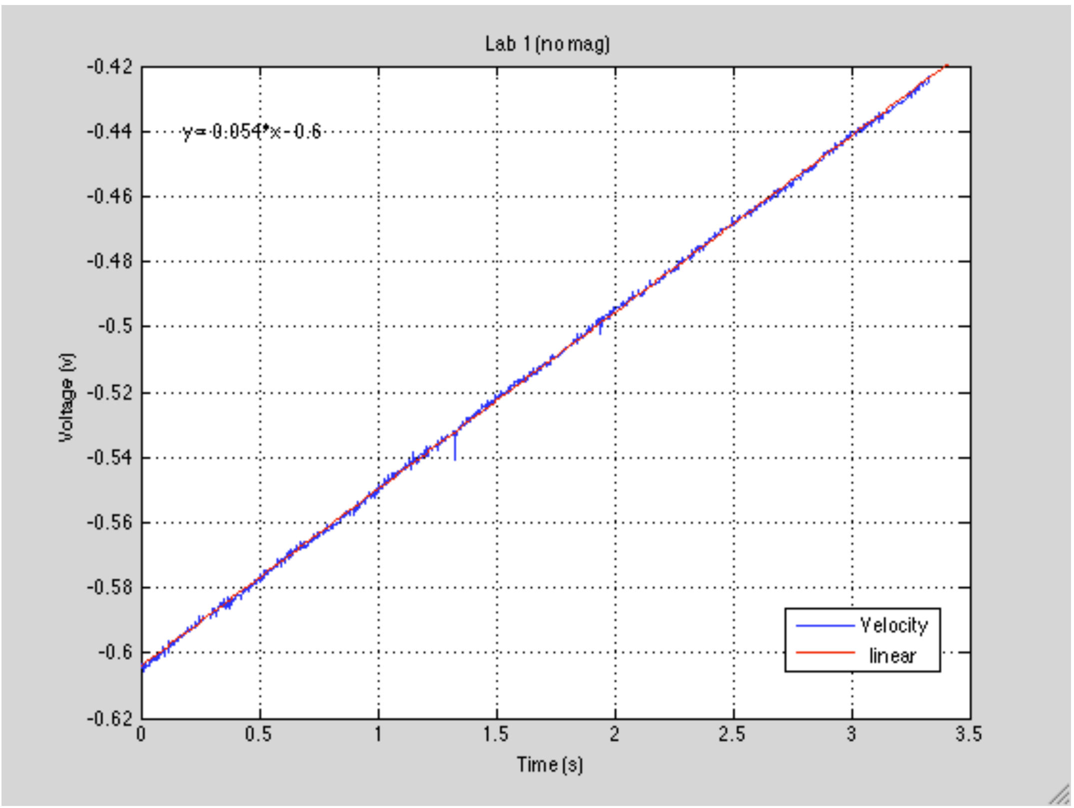
- Generate clearly labeled plots and indicate, for each case, which kind of friction dominate the damping behavior.

Save your data so that it is readily available later (online or on USB.)

You will need them for Problem Set 1.

Making A Good Plot with MATLAB

- **Put a meaningful title**
- **Label each axis (with proper unit)**
- **Label each data source**



- **<u>MATLAB commands</u>:**

```
>> mydata;
>> plot(t_mydata,v_mydata);
>> title('Lab 1 (no mag)');
>> xlabel('Time (s)');
>> ylabel('Voltage(V)');
>> legend('Velocity','Linear');
>> grid;
>>
```

Solving the equations of motion

MITMECHE

- For pure Coulomb damping: $J\dot{\omega}(t) = T_{ext}(t) - T_c(t)$

  (The external torque is "0" for our experiment, $T_c(t)$ is relatively constant T_c)

  Solving this ODE $\Longrightarrow \boxed{\omega(t) = \omega_0 - \frac{T_c}{J}t}$

- For pure viscous damping: $J\dot{\omega}(t) + b\omega(t) = T_{ext} \Longrightarrow \dot{\omega}(t) + \frac{b}{J}\omega(t) = 0$

  Solving this ODE $\Longrightarrow \boxed{\omega(t) = \omega_0 e^{-(\frac{b}{J})t}}$

Fitting an Exponential Function…

- Estimate τ from the time at which the response has decayed to:

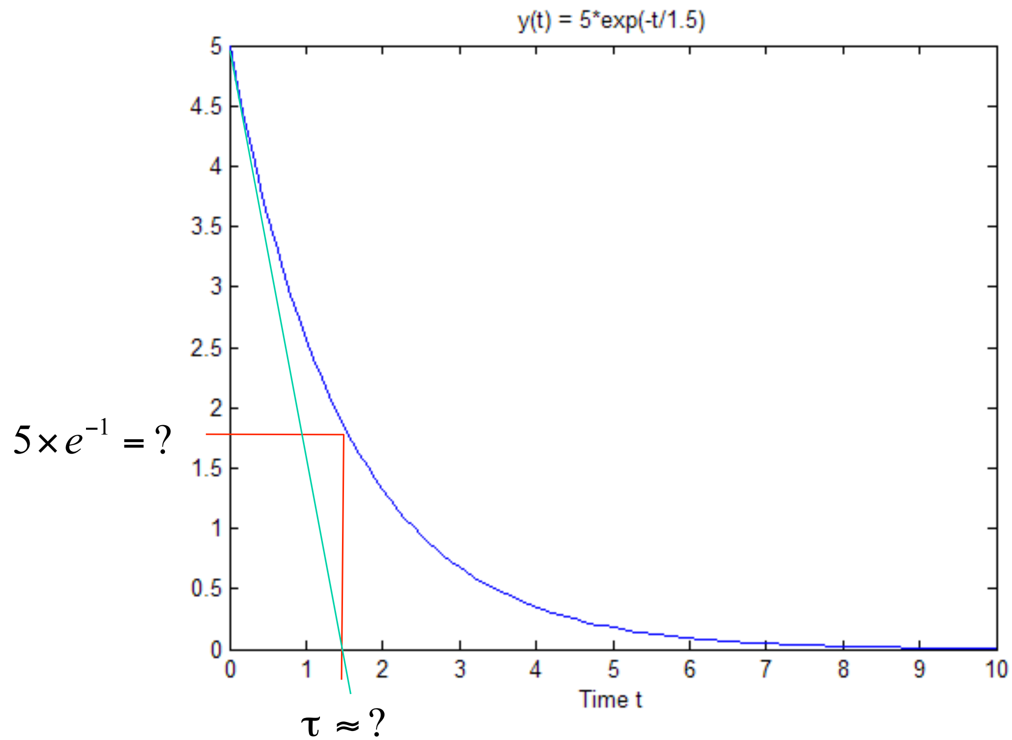
$$t = \tau$$

$$\boxed{y(t) = Ae^{-t/\tau}}$$

$$e^{-1} = ?$$

- Slope method

$$\boxed{\frac{dy}{dt} = -\frac{A}{\tau}e^{-t/\tau}}$$



Fitting an Exponential Function…

- **Log linear fit:**

$$y(t) = Ae^{-t/\tau}$$

$$\ln(y(t)) = \ln(A) + \left(\frac{-t}{\tau}\right)$$