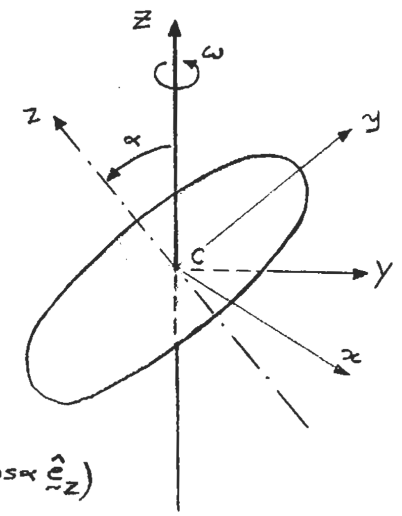
2.032 Dynamics

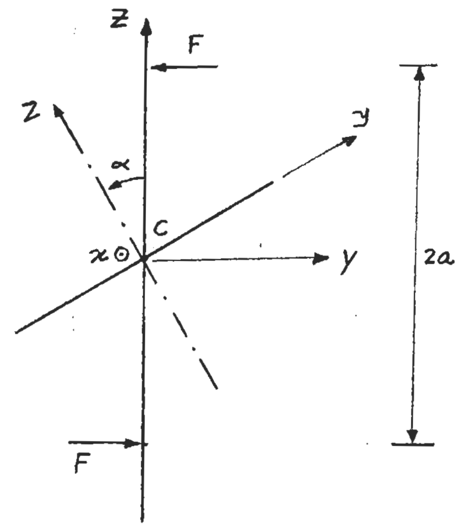
Fall 2003

Quiz No. 1

Problem 1

xyz coordinate system is fixed to the disk and rotates with ω about Z axis:





(a)

$$\underset{\sim}{\omega}_{disk} = \omega \underset{\sim}{\hat{e}}_Z = \omega\left(\sin\alpha\, \underset{\sim}{\hat{e}}_y + \cos\alpha\, \underset{\sim}{\hat{e}}_z\right)$$

$$\Longrightarrow \begin{cases} \omega_x = 0 \\ \omega_y = \omega \sin\alpha \\ \omega_z = \omega\cos\alpha \end{cases}$$

$$I_x = I_y = \frac{1}{4}MR^2 , \qquad I_z = \frac{1}{2}MR^2 \qquad \Longrightarrow \qquad [I]_C = \frac{1}{4}MR^2 \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ & & 2 \end{bmatrix}$$

$$\underset{\sim}{H}_C = [I]_C\, \underset{\sim}{\omega} = \frac{1}{4}MR^2 \begin{bmatrix} 1 & 0 & 0 \\ & 1 & 0 \\ & & 2 \end{bmatrix} \begin{Bmatrix} 0 \\ \omega\sin\alpha \\ \omega\cos\alpha \end{Bmatrix} = \boxed{\frac{1}{4}MR^2\omega\left(\sin\alpha\, \underset{\sim}{\hat{e}}_y + 2\cos\alpha\, \underset{\sim}{\hat{e}}_z\right)}$$

Angular momentum of the disk about C

(b) Since $\underset{\sim}{v}_c = 0$, forces on the bearings are equal and in opposite directions.

$$\underset{\sim}{\tau}_c = \frac{d\underset{\sim}{H}_c}{dt} = \frac{1}{4}MR^2\omega\left(\sin\alpha\, \underbrace{\frac{d\underset{\sim}{\hat{e}}_y}{dt}}_{\omega \underset{\sim}{\hat{e}}_Z \times \underset{\sim}{\hat{e}}_y = -\omega\cos\alpha\, \underset{\sim}{\hat{e}}_x} + 2\cos\alpha\, \underbrace{\frac{d\underset{\sim}{\hat{e}}_z}{dt}}_{\omega \underset{\sim}{\hat{e}}_Z \times \underset{\sim}{\hat{e}}_z = \omega\sin\alpha\, \underset{\sim}{\hat{e}}_x}\right)$$

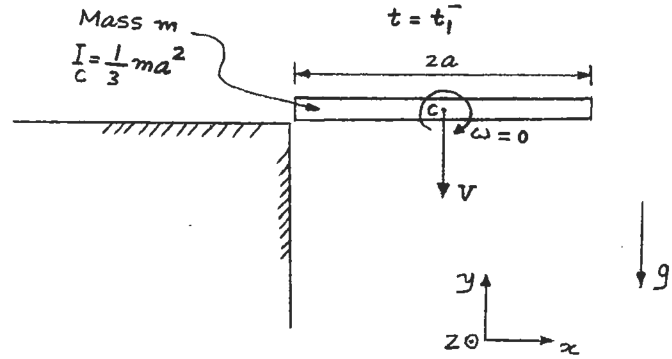
$$\therefore \quad 2aF\, \underset{\sim}{\hat{e}}_x = \frac{1}{4}MR^2\omega^2 \sin\alpha\cos\alpha\, \underset{\sim}{\hat{e}}_x$$

$$\Longrightarrow \boxed{F = \frac{MR^2\omega^2}{16a}\sin 2\alpha}$$ reaction forces at the bearings

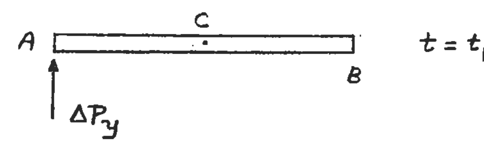
Note that force F rotates about Z axis with ω and is always in yz plane.

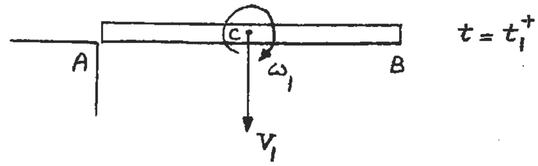
Problem 2

Assume the collision occurs at $t = t_1$:



A vertical impulse acts on the end A of the rod at $t = t_1$ (ΔP_y).
As a result, velocity of the center of mass would be V_1 and angular velocity ω_1 just after the impact.





(a) Energy is conserved in the collision: $KE + PE\big|_{t=t_1^-} = KE + PE\big|_{t=t_1^+}$

Gravity does not have enough time to act:

$$PE\big|_{t=t_1^-} = PE\big|_{t=t_1^+}$$

$$KE = \frac{1}{2} m v_C^2 + \frac{1}{2} I_C \omega^2 \qquad \left(I_C = \frac{1}{3} m a^2\right)$$

$$KE\big|_{t=t_1^-} = \frac{1}{2} m V^2 , \qquad KE\big|_{t=t_1^+} = \frac{1}{2} m V_1^2 + \frac{1}{6} m a^2 \omega_1^2$$

$$\therefore \quad \underline{V_1^2 + \frac{1}{3} a^2 \omega_1^2 = V^2} \qquad (1)$$

Angular momentum about point A on the table: $\underset{\sim}{\tau}_A = \frac{d}{dt} \underset{\sim}{H}_A + \cancel{\underset{\sim}{v}_A}^{\,0} \times \underset{\sim}{P}$

Again, gravity does not have time to act. $\longrightarrow$ $\underset{\sim}{\tau}_A = \underset{\sim}{0}$ $\longrightarrow$ $\frac{d}{dt} \underset{\sim}{H}_A = \underset{\sim}{0}$

$$\Longrightarrow \quad \underset{\sim}{H}_A\big|_{t=t_1^-} = \underset{\sim}{H}_A\big|_{t=t_1^+} \qquad \left(\underset{\sim}{H}_A = \underset{\sim}{H}_C + \underset{\sim}{AC} \times m \underset{\sim}{v}_C\right)$$

$$H_A\big|_{t=t_1^-} = -maV \, \underset{\sim}{\hat{e}}_z , \qquad \underset{\sim}{H}_A\big|_{t=t_1^+} = -maV_1 \, \underset{\sim}{\hat{e}}_z - \frac{1}{3} m a^2 \omega_1 \, \underset{\sim}{\hat{e}}_z$$

Problem 2

$$\therefore \quad \underset{\sim}{v_1 + \tfrac{1}{3} a\omega_1 = V} \qquad (2)$$

$$(1), (2) \Rightarrow \begin{cases} \cancel{v_1 = V}, & \cancel{\omega_1 = 0} \\ v_1 = \dfrac{V}{2}, & \omega_1 = \dfrac{3V}{2a} \end{cases}$$

angular velocity of the rod just after the impact.

(b)

$$\underset{\sim}{v}_{A\,rod}\Big|_{t=t_1^+} = \underset{\sim}{v}_C\Big|_{t=t_1^+} + \underset{\sim}{\omega}_{rod}\Big|_{t=t_1^+} \times \underset{\sim}{CA}$$

$$= -V_1\,\hat{\underset{\sim}{e}}_y + (-\omega_1\,\hat{\underset{\sim}{e}}_z) \times (-a\,\hat{\underset{\sim}{e}}_x)$$

$$= -\frac{V}{2}\,\hat{\underset{\sim}{e}}_y + \left(-\frac{3V}{2a}\,\hat{\underset{\sim}{e}}_z\right) \times (-a\,\hat{\underset{\sim}{e}}_x) = \left(-\frac{V}{2} + \frac{3V}{2}\right)\hat{\underset{\sim}{e}}_y = V\,\hat{\underset{\sim}{e}}_y$$

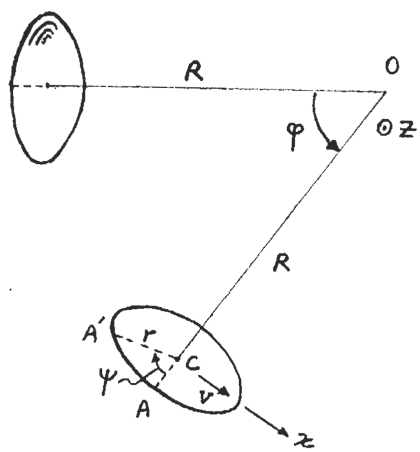
Velocity of the end of the rod that touched the table, just after the impact.

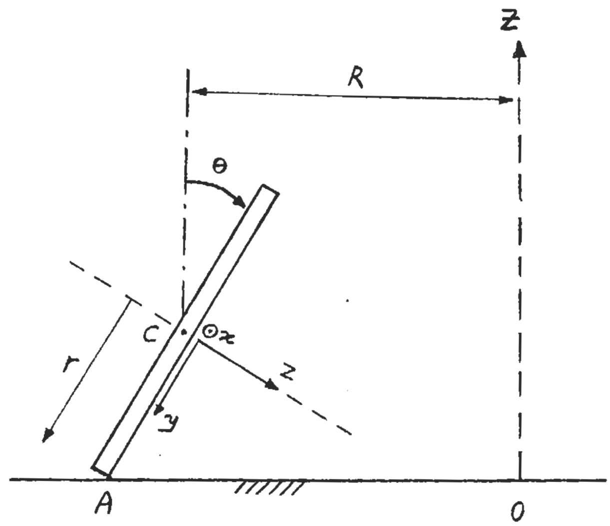
$$\underset{\sim}{v}_{A\,rod}\Big|_{t=t_1^-} = -V\,\hat{\underset{\sim}{e}}_y$$ just before the impact.

The result seems reasonable. Magnitude of the velocity of A on the rod is conserved and $\underset{\sim}{v}_{A\,rod}$ just changes the direction in the collision. This shows an elastic collision which is expected when energy is conserved.

Problem 3

Assume thickness of the disk is small relative to its radius r.





xyz coordinate system rotates about Z axis.

$$\underset{\sim}{v}_C = V\,\hat{\underset{\sim}{e}}_x = R\dot{\varphi}\,\hat{\underset{\sim}{e}}_x \qquad \underset{\sim}{\omega}_{disk} = \dot{\varphi}\,\hat{\underset{\sim}{e}}_Z + \dot{\psi}\,\hat{\underset{\sim}{e}}_z = \overset{\frac{V}{R}}{\dot{\varphi}}\left(-\cos\theta\,\hat{\underset{\sim}{e}}_y - \sin\theta\,\hat{\underset{\sim}{e}}_z\right) + \dot{\psi}\,\hat{\underset{\sim}{e}}_z$$

No slip. $\longrightarrow$ $\underset{\sim}{v}_{A\ disk} = \underset{\sim}{0}$

$$\underset{\sim}{v}_{A\ disk} = \underset{\sim}{v}_C + \underset{\sim}{\omega}_{disk} \times \underset{\sim}{CA}$$

$$= V\,\hat{\underset{\sim}{e}}_x + \left[-\frac{V}{R}\cos\theta\,\hat{\underset{\sim}{e}}_y + \left(\dot{\psi} - \frac{V}{R}\sin\theta\right)\hat{\underset{\sim}{e}}_z\right] \times \left(r\,\hat{\underset{\sim}{e}}_y\right)$$

$$= \left[V - \left(\dot{\psi} - \frac{V}{R}\sin\theta\right)r\right]\hat{\underset{\sim}{e}}_x = \underset{\sim}{0}$$

$$\therefore \quad \dot{\psi} = V\left(\frac{1}{r} + \frac{1}{R}\sin\theta\right)$$

$$\Longrightarrow \quad \underset{\sim}{\omega}_{disk} = -\frac{V}{R}\cos\theta\,\hat{\underset{\sim}{e}}_y + \frac{V}{r}\,\hat{\underset{\sim}{e}}_z$$

Angular velocity of the disk