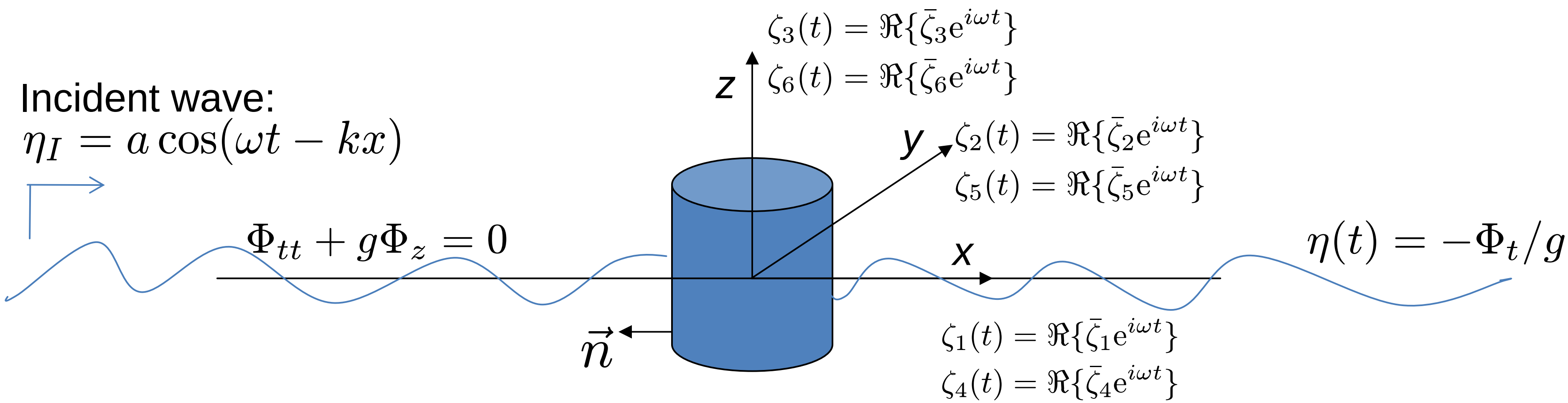
2.019 Design of Ocean Systems

Lecture 8

Seakeeping (IV)

March 4, 2011

General Response of A Floating Body in Regular Ambient Waves



Equation of motion: $$\sum_{\ell=1}^{6}[(M_{j\ell} + A_{j\ell})\ddot{\zeta}_\ell + B_{j\ell}\dot{\zeta}_\ell + C_{j\ell}\zeta_\ell] = \bar{F}_{Ej}e^{i\omega t} \quad (j = 1, \ldots, 6) \qquad (1)$$

$$\longrightarrow \sum_{\ell=1}^{6}[-\omega^2(M_{j\ell} + A_{j\ell}) + i\omega B_{j\ell} + C_{j\ell}]\bar{\zeta}_j = F_{Ej} \quad (j = 1, \ldots, 6)$$

$M_{j\ell}$: 6×6 elements of the egeneralized mass matrix

$A_{j\ell}, B_{j\ell}$: 6×6 elements of added mass and wave damping matrices

$C_{j\ell}$: 6×6 elements of hydrostatic restoring matrix

Transfer function or Response Amplitude Operator (RAO): $H_j(\omega) = \frac{\bar{\zeta}_j(\omega)}{a} \quad (j = 1, \ldots, 6)$

Numerical Method for Potential-Flow Problems

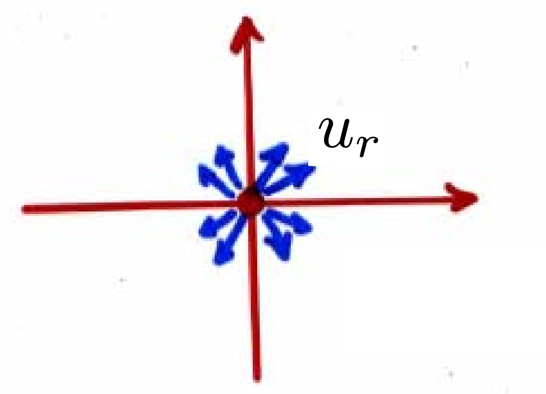
Uniform free stream:

$$\Phi = Ux \quad \rightarrow u = U, v = 0, w = 0$$

2D point source:

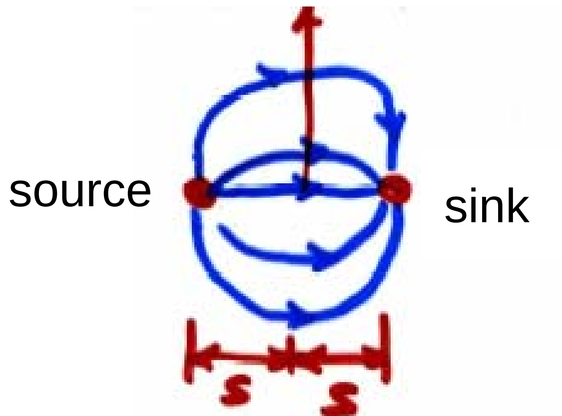
$$\Phi = \frac{m}{2\pi} \ln \sqrt{x^2 + z^2} = \frac{m}{2\pi} \ln r$$

$$u_r = \frac{m}{2\pi r}$$



2D point source plus point sink:

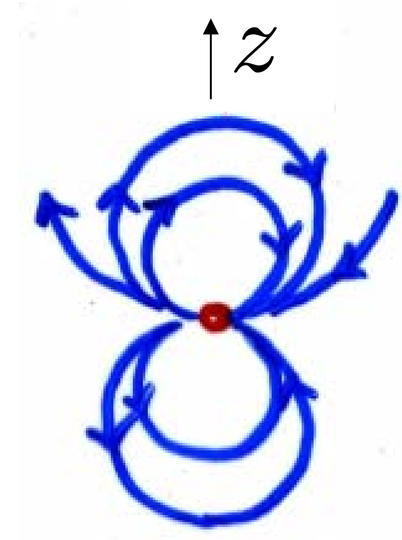
$$\Phi = \frac{m}{2\pi} \ln \sqrt{(x+s)^2 + z^2} - \frac{m}{2\pi} \ln \sqrt{(x-s)^2 + z^2}$$



2D doublet or dipole: source + sink, as $s \to 0$ while keeping $2ms = \mu$.

$$\Phi = \lim_{s\to 0} \frac{m}{2\pi} \ln \left\{ \frac{\sqrt{(x+s)^2 + z^2}}{\sqrt{(x-s)^2 + z^2}} \right\}$$

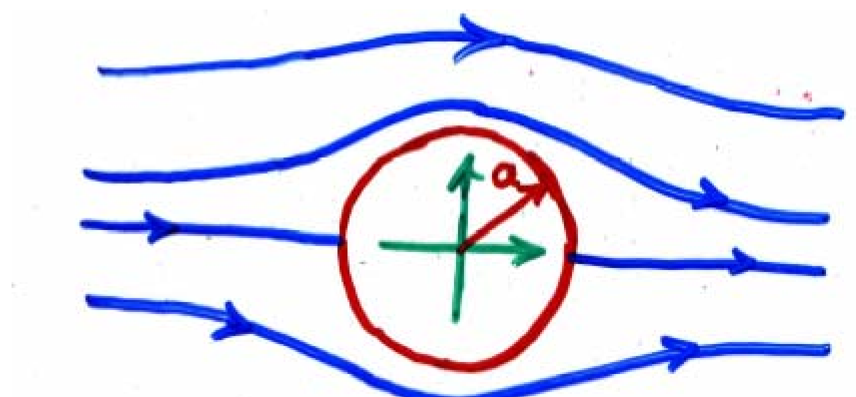
$$= \lim_{s\to 0} \frac{m}{2\pi} \frac{2xs}{\sqrt{x^2+z^2}} = \frac{\mu}{2\pi} \frac{x}{\sqrt{x^2+z^2}}$$



2D Stream plus dipole:

$$\Phi = Ux + \frac{\mu}{2\pi} \frac{x}{\sqrt{x^2+z^2}}$$

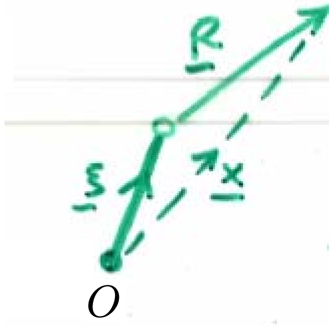
$$a = \sqrt{\frac{\mu}{2\pi U}}$$



Three-dimensional point source:

$$\Phi(\vec{x},\vec{\xi}) = -\frac{Q}{4\pi R}$$

$$= -\frac{Q}{4\pi}\frac{1}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}}$$



Three-dimensional source distribution:

Distribute sources of strength $M(\vec{\xi},t)\mathrm{d}S$, varying with space ξ and pulsating in time t and proportional to surface area $\mathrm{d}S$:

sources are distributed
over the surface of the body

$$\mathrm{d}\Phi = \frac{1}{4\pi}M(\vec{\xi},t)G(\vec{x},\vec{\xi})\mathrm{d}S$$

$G(\vec{x},\vec{\xi})$: Green's function

Green function in unbounded fluid:

$$G(\vec{x},\vec{\xi}) = \frac{1}{R} = \frac{1}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}}$$

Free-surface Green function (in finite depth H) satisfying linearized free-surface boundary condition:

$$\begin{aligned} G(\vec{x},\vec{\xi}) &= \frac{1}{R} + \frac{1}{R'} \\ &= +2\int_0^\infty \frac{(\mu+\nu)e^{-\mu H}}{\mu \sinh \mu H - \nu \cosh \nu H} \cosh \mu(\zeta + H) \cosh \mu(z+H) J_0(\mu r) \mathrm{d}\mu \\ &+2\pi \mathrm{i} \frac{k^2-\nu^2}{(k^2-\nu^2)H+\nu} \cosh k(z+H) \cosh k(\zeta+H) J_0(kr) \end{aligned}$$

$$\nu = \frac{\omega^2}{g} = k \tanh kH$$

$$R = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}, \qquad R' = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+2H+\zeta)^2}$$

Source Method

- Distribution sources on the body surface with unknown strengths:

$$M(\vec{\xi}, t) = \mathrm{Re}\{\bar{M}(\vec{\xi})e^{i\omega t}\}$$

Diffraction problem: $\bar{M}_D(\vec{\xi})$

$$\bar{\Phi}_D(\vec{x}) = \frac{1}{4\pi}\int_S \bar{M}_D(\vec{\xi})G(\vec{x}, \vec{\xi})\mathrm{d}S$$

Radiation problem: $\bar{M}_j(\vec{\xi})$

$$\bar{\Phi}_j(\vec{x}) = \frac{1}{4\pi}\int_S \bar{M}_j(\vec{\xi})G(\vec{x}, \vec{\xi})\mathrm{d}S$$

- The source strength is found by requiring the velocity satisfies the boundary condition on the body surface S

Boundary condition at $\vec{x} = \vec{x}_B$:

Diffraction problem:

$$-\frac{1}{2}\bar{M}_D(\vec{x}) + \frac{1}{4\pi}\fint_S \bar{M}_D(\vec{\xi})\frac{\partial}{\partial n}G(\vec{x}, \vec{\xi})\mathrm{d}S = -\frac{\partial \bar{\Phi}_I}{\partial n}$$

Radiation problem:

$$-\frac{1}{2}\bar{M}_j(\vec{x}) + \frac{1}{4\pi}\fint_S \bar{M}_j(\vec{\xi})\frac{\partial}{\partial n}G(\vec{x}, \vec{\xi})\mathrm{d}S = -(i\omega)n_j$$

- To solve the integral equation for unknown source strengths, we apply the so-called panel method: Subdividing the body surface into N elements with the assumption of an uniform distribution of source strength over each element. This will leads to N equations and N unknown source strengths:

$$-\bar{M}_D(\vec{x}_m) + \sum_{n=1}^{N} \alpha_{mn} \bar{M}_D(\vec{x}_n) = -\frac{\partial \bar{\Phi}_I(\vec{x}_m)}{\partial n}$$

$$m = 1, 2, \cdots, N$$

$$\alpha_{mn} = \int_{\Delta S_n} \frac{\partial}{\partial n} G(\vec{x}_m, \vec{\xi}_n) \mathrm{d}S$$

- Once unknown source strengths on the body are found, the diffraction and radiation potentials can be evaluated:

$$\bar{\Phi}_D(\vec{x}) = \sum_{n=1}^{N} \bar{M}_D(\vec{x}_n) \frac{1}{4\pi} \int_{\Delta S_n} G(\vec{x}, \vec{\xi}_n) \mathrm{d}S$$

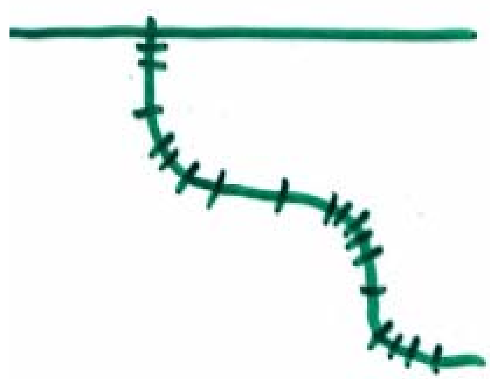
- Numerical solution of the linear system of *N* equations:

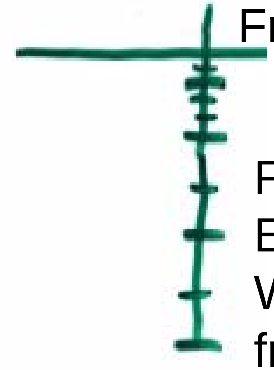
$$[A]\{M\} = \{b\}$$

Gauss elimination $\sim O(N^3)$ computational effort
Iteration solver $\sim O(N^2)$ computational effort

Convergence with error $\sim 1/N$ as $N \rightarrow \infty$

For better convergence of the solution, discretization must be fine where geometry changes sharply or near free surface.





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Please see Fig. 3 and Fig. 5 from Newman, J., and P. Sclavounos. "The Computation of Wave Loads on Large Offshore Structures."

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