41 Floating Structure Heave and Roll

Consider the heave and roll response of a two-hull structure with the parameters below:

mass (metric tons)	3000
body rotary inertia (kg-m^2)	3×10^8
beam of each hull b (m)	5
open space between hulls B (m)	20
draft T (m)	10
vessel length L (m)	30
each hull effective added mass A_{33} (tons)	750
each hull effective damping coefficient B_{33} (Ns/m)	4×10^5

The hull is assumed to be uniform in cross-section over the entire length. We are going to study the behavior of this structure in beam waves - that is, moving from port to starboard across the two hulls. We will use the Bretschneider spectrum as in Homework 2, for sea states 2-6.

1. About what are the undamped natural frequencies in heave and in roll? Assume in this problem that the vessel's center of mass is near the waterline.

   The square root of waterplane stiffness divided by mass and added mass gives about 0.700 rad/s in heave and 0.875 rad/s in roll.

2. About what are the damping ratios in heave and in roll?

   We have $\zeta \simeq 0.095$ in heave and 0.12 in roll; not much damping!

3. Up to what wave frequency is the long-wavelength approximation valid for this problem? Make a plot to verify that you have a ratio of at least three or so, up to the highest frequency you will use in the following calculations.

   The figure below shows that we are OK up to about two radians per second, considering each hull alone. If we take the whole vessel beam, then we could only do the higher sea states, e.g., $\omega < 0.9$.

4. Taking $y = 0$ at the port side of the port hull, compute for a range of wave frequencies the phase angle in the incident wave heave force (F_{3I} in the notes) seen at the middle of the port hull and then the middle of the starboard hull. Make and annotate a plot.

   This is shown in the figure; formulas for each term are given below. The angle for the far hull changes very quickly as the wavelengths grow to and then exceed B.

5. Write out the differential equation that governs the heave motions, under wave disturbances, and including the incident wave, diffraction, and radiation terms.

We have from the formulas in the notes

$$\begin{aligned} mz'' + (B_{33p} + B_{33s})z' + (C_{33p} + C_{33s})z &= (F_{3Ip} + F_{3Is})\eta(y=0) + \\ &\quad (F_{3Dp} + F_{3Ds})\eta(y=0) + (F_{3Rp} + F_{3Rs})z \end{aligned}$$

where the p and s subscripts refer to port and starboard hulls, respectively. The coefficients are:

$$\begin{aligned} F_{3Ip} &= -\rho g \frac{e^{-kT}}{k}[\sin \omega t(\cos kb - \cos 0) - \cos \omega t(\sin kb - \sin 0)] \\ F_{3Is} &= -\rho g \frac{e^{-kT}}{k}[\sin \omega t(\cos k(2b+B) - \cos k(b+B)) - \cos \omega t(\sin k(2b+B) - \\ &\quad \sin k(b+B))] \\ F_{3Dp} &= -e^{-kT/2}A_{33p}\omega^2[\cos \omega t \cos(kb/2) + \sin \omega t \sin(kb/2)] \\ F_{3Ds} &= -e^{-kT/2}A_{33s}\omega^2[\cos \omega t \cos(k(B+b+b/2)) + \sin \omega t \sin(k(B+b+b/2))] \\ F_{3Rp} &= A_{33p}\omega^2 \\ F_{3Rs} &= A_{33s}\omega^2 \end{aligned}$$

6. Write the transfer function for heave, relating the input (wave elevation at $y = 0$) to the output (heave motion). You have to separate the time and space components, because the two hulls are in different locations: you should see terms like $\sin(\omega t)$, $\sin(k(b + B + b/2))$ and so on. Note that k here is related to ω through the dispersion relation. In the frequency domain, $\cos \omega t$ will become simply one, and $\sin \omega t$ will become $-j = -\sqrt{-1}$.

   Collecting the terms in z to the left-hand side of the above equation, and η on the right, we get

$$\frac{z(\omega)}{\eta(x=0,\omega)} = \frac{F_{3Ip} + F_{3Is} + F_{3Dp} + F_{3Ds}}{-F_{3Rp} - F_{3Rs} - m\omega^2 + j\omega(B_{33s} + B_{33p}) + C_{33p} + C_{33s}}$$

7. Write out the differential equation that governs the roll motions, under wave disturbances, and including the incident wave, diffraction, and radiation terms.

   Using the coefficients above, we have

$$\begin{aligned} J\theta'' + (B_{33p} + B_{33s})r^2\theta' + (C_{33p} + C_{33s})r^2\theta &= (F_{3Ip} - F_{3Is} + F_{3Dp} - F_{3Ds})r\eta(x=0) + \\ &\quad (F_{3Rp} - F_{3Rs})r^2\theta \end{aligned}$$

   Here r is the moment arm from the centerline to the middle of each hull, i.e., $r = B/2 + b/2$.

8. Write the transfer function for roll, relating the input (wave elevation at $y = 0$) to the output (roll motion).

Collecting terms again, the roll transfer function is:

$$\frac{\theta(\omega)}{\eta(x=0,\omega)} = \frac{(F_{3Ip} - F_{3Is} + F_{3Dp} - F_{3Ds})r}{-(F_{3Rp} - F_{3Rs})r^2 - \omega^2 J + j\omega(B_{33p} + B_{33s})r^2 + (C_{33p} + C_{33s})r^2}$$

9. Make a plot of the two transfer function magnitudes, showing the heave transfer function in units of (meters/meter) and the roll in (degrees/meter).

   See attached plot.

10. How do the major features in the transfer function relate to the phase plot you made? How do they relate to the natural frequencies and damping ratios you estimated?

    First consider the undamped natural frequencies in heave and roll, which we found to be about 0.7 and 0.87 rad/s respectively. There is a peak in the heave transfer function near 0.75, and a peak in roll around 1.0; these are in reasonable agreement. More interestingly, referring to the phase plot, we know that the incident forcing on the hulls is out of phase at about 1.1 rad/s and this is a point where the transfer function in heave is near zero, while in roll we have a big response. Note also how close this point is to the roll resonance. On the other hand, at about 1.6 rad/s the incident forcing is in phase (360 degrees) across the two hulls; at this point, the heave transfer function is reasonably large, while the roll transfer function goes to zero.

11. For each of the sea states [2-6], compute the "significant height" in of the vessel motion in heave (meters) and roll (meters).

    Square the transfer function and multiply it by the spectrum to get the spectrum of the response. See the figure, which shows results for all the sea states. I get significant heights in heave of [0.076, 0.32, 0.75, 1.26,1.52] meters, and in roll of [4.3, 18, 43, 72, 87] degrees. One should question the linear assumptions for such large roll angles as this!

```
%-------------------------------------------------------------------
% heave and pitch analysis of a pair of hulls in waves
%

clear all;

g = 9.81 ; % gravity
rho = 1000 ; % water density

Tu = 5 ; % draft of each hull, upper
bu = 5 ; % beam of each hull, uppper
Tl = 10 ; % draft of lower hull
bl = 5 ; % beam of lower hull
B = 20 ; % distance between the two hulls' inner faces
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```
L = 30 ; % "length" of the pair of hulls

% show the data from the table in the problem
SSvec = [2 3 4 5 6] ; % sea states
wmvec = 2*pi*ones(size(SSvec)) ./ [6.3 7.5 8.8 9.7 12.4] ;
    % modal frequencies
Hsigvec = [0.3 0.9 1.9 3.3 5.0] ; % significant wave heights

arm = (B/2 + bl/2) ; % moment arm for stiffness and damping terms

Del = 2*L*( Tu*bu + (Tl-Tu)*bl ) ; % volume
m = Del*rho ; % mass
J = m*(B/2)^2 ; % rotary moment of inertia of the body, approx.

A33s = rho*bl^2*L ; % added mass, approx.
A33p = A33s ;

B33s = 1/2*rho*L*bl*8/3/pi*(2*pi/10)*10 ;
B33p = B33s ;

C33s = rho*g*bu*L ; % waterplane area stiffness, stbd hull
C33p = C33s ;       % port hull

[m J bu B Tl L A33s B33s]' % show us the matrix of parameters

disp('---------------------------------------------------------------');
disp(sprintf('Approximate heave nat. freq:  %g rad/s', ...
    sqrt(C33s/(m)))); % with added mass
disp(sprintf('Approximate roll nat. freq:  %g rad/s', ...
    sqrt(C33s*arm^2/J))); % with added mass
disp(sprintf('Approximate damping ratio in heave:  %g', ...
    B33s / 2 / sqrt(m*C33s))); % with added mass
disp(sprintf('Approximate damping ratio in roll:  %g', ...
    B33s*arm^2 / 2 / sqrt(J*C33s*arm^2))); % with added mass

sinwt = -sqrt(-1) ; % frequency domain equivalents for sin and cos
coswt = 1 ;

wvec = .02:.01:2 ;

for i=1:length(wvec),
    w = wvec(i) ;

    k  = w^2 / g ; % dispersion relation
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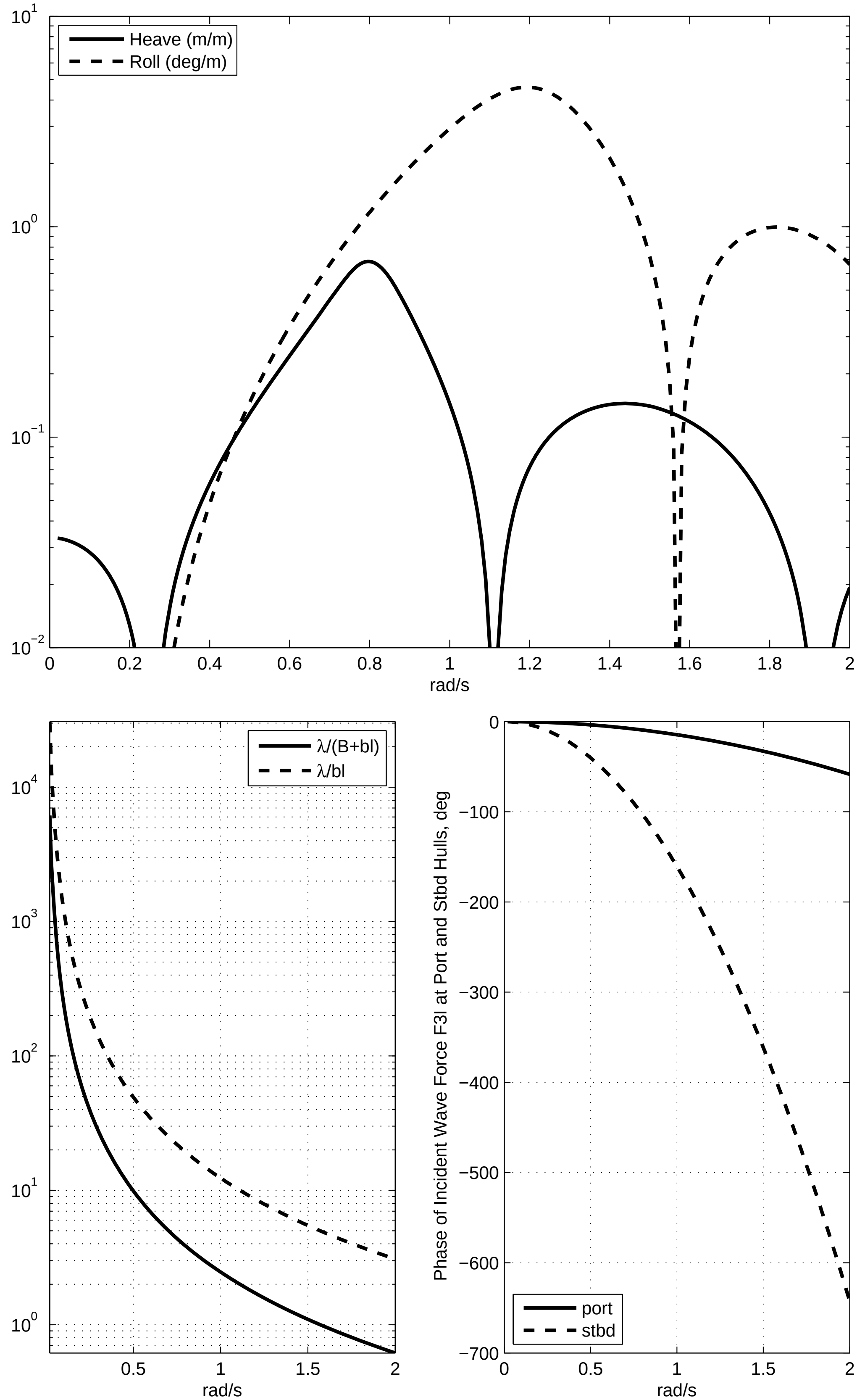
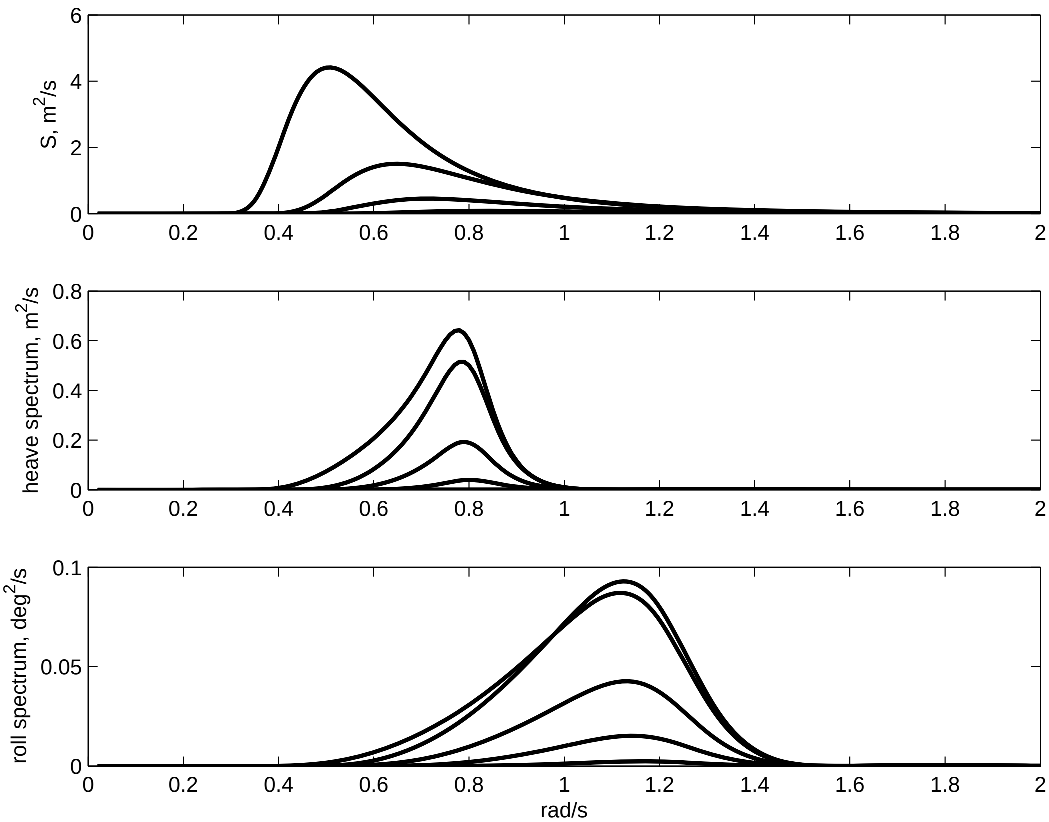
```
    lam(i) = 2*pi/k ; % wavelength

    % force due to incident waves, port and starboard.  We
    % take y=0 at the port side of the port hull
    F3Ip(i) = -rho*g*exp(-k*Tl)/k*[
            sinwt*(cos(k*bl)-cos(0)) - ...
            coswt*(sin(k*bl)-sin(0))] ;
            % multiplies wave elevation at y=0
    F3Is(i) = -rho*g*exp(-k*Tl)/k*[
            sinwt*(cos(k*(2*bl+B))-cos(k*(bl+B))) - ...
            coswt*(sin(k*(2*bl+B))-sin(k*(bl+B)))] ;
            % multiplies wave elevation at y=0

    % forces due to diffraction
    F3Dp(i) = exp(-k*Tl/2)*A33p*(-w^2)*[coswt*cos(k*bl/2) + ...
        sinwt*sin(k*bl/2)] ;
            % multiplies wave elevation at y=0
    F3Ds(i) = exp(-k*Tl/2)*A33s*(-w^2)*[coswt*cos(k*(B+bl+bl/2)) + ...
            sinwt*sin(k*(B+bl+bl/2))] ;
            % multiplies wave elevation at y=0

    % forces due to radiation
    F3Rp(i) = -A33p*(-w^2) ; % multiplies body heave at port hull
    F3Rs(i) = -A33s*(-w^2) ; % multiplies body heave at stbd hull

    tfHeave(i) = (F3Ip(i) + F3Is(i) + F3Dp(i) + F3Ds(i)) / ...
        (-F3Rp(i) - F3Rs(i) - m*w^2 + ...
        sqrt(-1)*w*(B33s + B33p) + (C33s + C33p)) ;

    tfRoll(i) = ...
        ( F3Ip(i)*arm - F3Is(i)*arm + F3Dp(i)*arm - F3Ds(i)*arm) / ...
        (-F3Rp(i)*arm^2 + F3Rs(i)*arm^2 - J*w^2 + ...
        sqrt(-1)*w*(B33p*arm^2 + B33s*arm^2) + ...
        (C33p*arm^2 + C33s*arm^2)) ;

end;

figure(1);clf;hold off;
semilogy(wvec,abs(tfHeave),wvec,(abs(tfRoll)*180/pi),'--','LineWidth',2);
axis([0 2 .01 10]);
legend('Heave (m/m)','Roll (deg/m)',2);
xlabel('rad/s');

figure(2);clf;hold off;
```

```
subplot(122)
plot(wvec,unwrap(angle(F3Ip))*180/pi,wvec,...
    unwrap(angle(F3Is))*180/pi,'--','LineWidth',2);
ylabel('Phase of Incident Wave Force F3I at Port and Stbd Hulls, deg');
xlabel('rad/s');
grid;
legend('port','stbd',3);
subplot(121);
semilogy(wvec,lam/(B+bl),wvec,lam/bl,'--','LineWidth',2) ;
grid;legend('\lambda/(B+bl)','\lambda/bl');
axis('tight');
xlabel('rad/s');

% MAKE UP THE BRETSCHNEIDER SPECTRUM

figure(4);clf;hold on;
% step through the different seastates
for i = 1:length(wmvec),

wm = wmvec(i) ;
Hsig = Hsigvec(i) ;
SS = SSvec(i) ;

for j = 1:length(wvec),
w = wvec(j) ;
S(j) = 5/16 * wm^4 / w^5 * Hsig^2 * exp(-5 * wm^4 / 4 / w^4) ;
end;

% check that we got the right formula!
disp(sprintf(...
        '[Square Root of Integral of Area of S:  %g;  Hsig/4:  %g]', ...
sqrt(sum(S)*mean(diff(wvec))), 1/4*Hsig));

    Heave = S.*conj(tfHeave).*tfHeave ;
    Roll = S.*conj(tfRoll).*tfRoll ;

    subplot(311);
    plot(wvec,(S),'LineWidth',2);
    ylabel('S, m^2/s');
    hold on;
%    a=axis ; axis([a(1:2) .001 3]);
    subplot(312);
    plot(wvec,(abs(Heave)),'LineWidth',2);
    hold on;
```

```
%    a=axis ; axis([a(1:2) .001 10]);
    ylabel('heave spectrum, m^2/s');
    subplot(313);
    plot(wvec,(abs(Roll))*180/pi,'LineWidth',2);
    hold on;
%    a=axis ; axis([a(1:2) .001 .5]);
    xlabel('rad/s');
    ylabel('roll spectrum, deg^2/s');

    sigHeaveHeight = sqrt(sum(Heave)*mean(diff(wvec)))*4 ;
    sigRollHeight = sqrt(sum(Roll)*mean(diff(wvec)))*4 ;

    disp(sprintf('For SeaState %d:',SS));
    disp(sprintf('The significant height in heave is %g m.',...
        sigHeaveHeight));
    disp(sprintf('The significant height in  roll is %g deg.',...
        sigHeaveHeight*180/pi));

end;
%-----------------------------------------------------------------------
```


Heave (m/m)
Roll (deg/m)
rad/s
λ/(B+bl)
λ/bl
rad/s
Phase of Incident Wave Force F3I at Port and Stbd Hulls, deg
port
stbd
rad/s


S, m^2/s
heave spectrum, m^2/s
roll spectrum, deg^2/s
rad/s