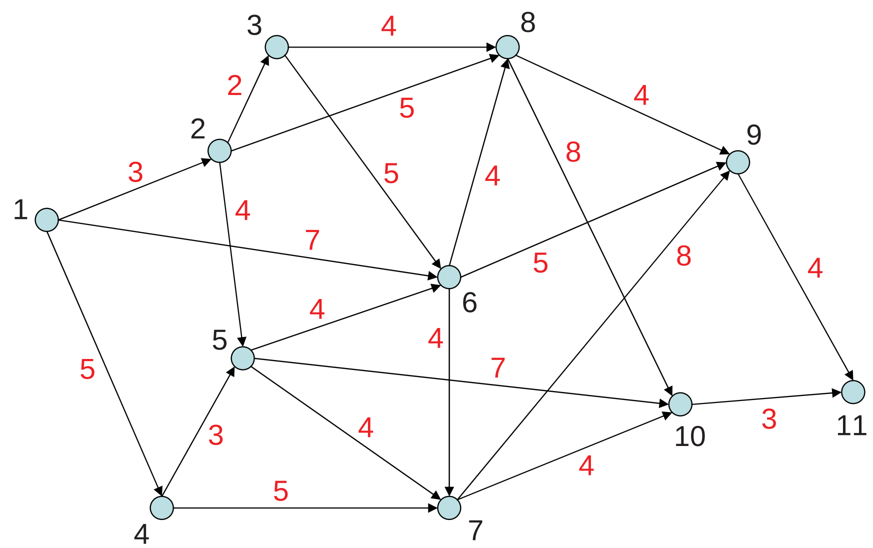
30 Dynamic Programming for Path Design

Given the transition costs in red, what are the maximum and minimum costs to get from node 1 to node 11? This situation is encountered when planning paths for autonomous agents moving through a complex environment, e.g., a wheeled robot in a building.



Solution: The minimum cost is 16 (path [1,6,9,11] or [1,2,8,9,11]) and the maximum value is 28 (path [1,4,5,6,7,9,11]!). The attached code uses value iteration to find these in two and five iterations, respectively.

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% Value iteration solution of deterministic dynamic programming.
% The program looks complicated only because I cover both
% minimization and maximization in the same program!

clear all;

ch = input('Find minimum (0) or maximum (1):  ');
if ~ch,
    init = 1e6 ;    % look for minimum;
                    % big initial guesses for costs to go
else,
    init = 1e-6 ;   % look for maximum;
                    % small initial guesses for value to go
end;

% interconnect matrix:  row is the node (first is starting
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% point) and column is the set of nodes pointed to.  Note
% that the ending node is not included because it points to
% nowhere.
I = [[2 6 4]        % node 1 (start) points to nodes 2,6,4
    [3 8 5]         % node 2 points to nodes 3,8,5.  And so on...
    [8 6 NaN]       % node 3
    [5 7 NaN]       % node 4
    [6 7 10]        % node 5
    [8 9 7]         % node 6
    [10 9 NaN]      % node 7
    [10 9 NaN]      % node 8
    [11 NaN NaN]    % node 9
    [11 NaN NaN]];  % node 10

% cost per link - these go with the interconnects in A.  Note
% that the entries with direct connection to the end node are NaN,
% because we will enforce the link cost in ctg (below) explicitly
C = [[3 7 5]        % The cost is 3 to move between nodes 1 and 2,
                    % and 7 to move between nodes 1 and 6, etc.
    [2 5 4]         % node 2
    [4 5 NaN]       % node 3
    [3 5 NaN]       % node 4
    [4 4 7]         % node 5
    [4 5 4]         % node 6
    [4 8 NaN]       % node 7
    [8 4 NaN]       % node 8
    [NaN NaN NaN]   % node 9
    [NaN NaN NaN]]; % node 10

% initial guess of cost-to-go (or value-to-go) at each node
tg = [[NaN]         % node 1
    [init]          % node 2
    [init]          % node 3
    [init]          % node 4
    [init]          % node 5
    [init]          % node 6
    [init]          % node 7
    [init]          % node 8
    [4]             % node 9 (points directly to end, node 11)
    [3]];           % node 10 (points directly to end, node 11)

w = size(I,2);  % width of interconnect matrix

disp(sprintf('%g ',tg));    % list the first cost-to-go or
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                             % value-to-go

for k = 1:5,   % carry out a fixed number of iterations

    % cycle through the nodes one by one.  Note that we don't
    % need to recompute tg for nodes that point to the end
    for i = 1:sum(~isnan(C(:,1))),

        % We'll look for the minimum estimated cost-to-go
        % (or maximum estimated value-to-go) across
        % the possible nodes pointed to
        if ~ch,
            dummy = 1e6 ;  % initialize to be huge
        else,
            dummy = 1e-6 ;  % initialize to be tiny
        end;

        % look at all the nodes pointed to from node i
        for j = 1:w,
            if ~isnan(I(i,j)), % consider only true entries in I

                test = tg(I(i,j)) + C(i,j) ;

                if ~ch,     % look for minimum
                    if test < dummy,
                        dummy = test ;
                    end;
                else,       % look for maximum
                    if test > dummy,
                        dummy = test ;
                    end;
                end;

            end;    % "true entries"
        end;    % j: nodes pointed to
        tg(i) = dummy ;
    end;    % i: nodes
    disp(sprintf('%g ',tg));
end;    % k: iteration

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