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1) Reynolds number $Re = \frac{\rho U L}{\mu}$

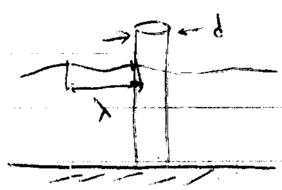
	U	L	D
submarine	30 kts ≈ 15 m/s	250 ft	20 ft
model		2.5 ft	0.2 ft

choose $\frac{1}{100}$ scale to fit in tank

$$Re_{sub} = Re_{model}$$

$$\frac{\rho \cdot 15\,m/s \cdot L}{\mu} = \frac{\rho\, U_m \frac{L}{100}}{\mu}$$

$U_m = 1500$ m/s ← This is very fast and will not be possible in the tank.

2. 

$\frac{\lambda}{d} = \frac{10}{0.5} = 20 \Rightarrow$ can be neglected

inertial force dominates

3. $F = \phi(d, f, V, \rho, \mu)$

$\frac{kg\,m}{s^2}$ $\quad m \quad \frac{1}{s} \quad \frac{m}{s} \quad \frac{kg}{m^3} \quad \frac{kg}{m\,s}$

$$\Pi_1 = \frac{F}{\rho V^2 d^2}, \quad \Pi_2 = \frac{\mu}{\rho V d} = \frac{1}{Re}, \quad \Pi_3 = \frac{fd}{V} = St$$

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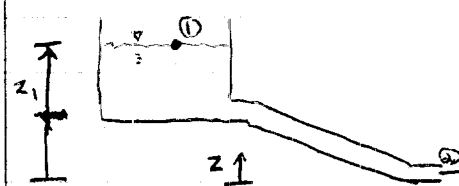
4) $Q = 45 \text{ survivors} \cdot \frac{0.0015\, m^3}{day} \cdot \frac{1\ day}{12\ hr} \cdot \frac{hr}{3600\, s} = 1.56 \times 10^{-6}\ m^3/s$

$A = \frac{\pi (0.1)^2}{4} = 0.0075\ m^2$ (Note: diameter not given in problem.)

$Q = V \cdot A \Rightarrow V = \frac{Q}{A} = 2.1 \times 10^{-4}\ m/s$



$V_2 \approx 2.1 \times 10^{-4}\ m/s$

$$\underbrace{p_1}_{p_a} + \frac{1}{2}\rho \underbrace{\cancel{V_1^2}}_{0} + \rho g z_1 = \underbrace{p_2}_{p_a} + \frac{1}{2}\rho V_2^2 + \rho g \underbrace{\cancel{z_2}}_{0}$$

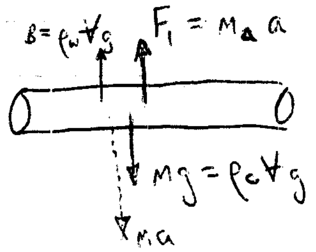
$$z_1 = \frac{V_2^2}{2g}$$

$$z_1 = 2.2 \times 10^{-9}\ m$$

5) ~~$J = \frac{Nd}{U} = \frac{\text{propeller tip speed}}{\text{ship speed}}$~~ $J = \frac{U}{Nd} = \frac{\text{ship speed}}{\text{prop. tip speed}}$

6) $St = \frac{fd}{U} \Rightarrow f = \frac{0.2 \cdot 4}{0.05} = 16\ Hz$

7)

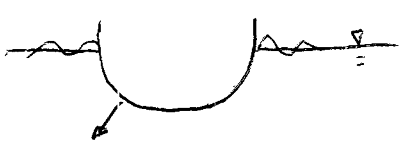


$$\underset{(down)}{\Sigma F = ma} = mg - B - F_1$$

$$\rho_c \forall a = \rho_c \forall g - \rho_w \forall g - \rho_w \forall a$$

$$\boxed{a = \frac{\rho_c - \rho_w}{\rho_c + \rho_w} g}$$ (down if positive)

8)



Laplace Eqn $\nabla^2 \phi_R = 0$

$\frac{\partial \phi}{\partial n} = \underline{V} \cdot \underline{n} \Big|_{body}$

$\phi_R \to 0$ far from body

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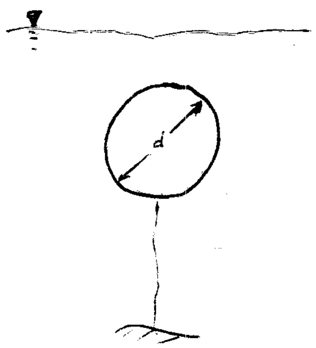
9) 

$$(m+m_a)\ddot{z} + B_{33}\dot{z} + C_{33}z = F(t)$$

$$m = \rho_a \forall = \rho_a \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$m_a = \frac{1}{2}\rho_w \forall = \rho_w \frac{2}{3}\pi\left(\frac{d}{2}\right)^3$$

$$C_{33} = 0$$

10)

$\boxed{1}$

$$\omega_n = \sqrt{\frac{k}{m+m_a}} = \sqrt{\frac{\rho_w \frac{\pi d^2}{4} g}{m + 0}}$$

$\boxed{2}$

$$\omega_n = \sqrt{\frac{\rho_w \frac{\pi d^2}{4} g}{m + \frac{2}{3}\rho_w \pi\left(\frac{D}{2}\right)^3}}$$

$\boxed{3}$

$$\omega_n = \sqrt{\frac{\rho_w \frac{\pi D^2}{4} g}{m + \frac{1}{3}\rho_w \pi\left(\frac{D}{2}\right)^3}}$$

$\boxed{3}$ should have least motion because "stiffest spring" (most buoyancy)

11) Since neutrally buoyant at any depth, there is no restoring force, and

$$\omega = \sqrt{\frac{K}{m+m_a}} = \sqrt{\frac{0}{m+m_a}} = 0$$

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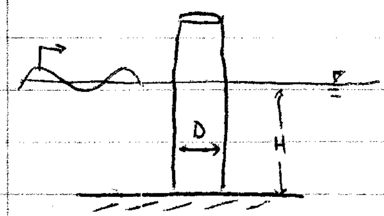
B1)



- deep water waves $\omega^2 = gk$
- phase speed $V_p = \frac{\omega}{k}$

a) $F = \phi(K, \omega, a, D, H, \rho, \mu, g)$

$\frac{kg\,m}{s^2}$ $\quad \frac{1}{m}$ $\quad \frac{1}{s}$ $\quad m$ $\quad m$ $\quad m$ $\quad \frac{kg}{m^3}$ $\quad \frac{kg}{ms}$ $\quad \frac{m}{s^2}$

$$\Pi_1 = \frac{F}{\rho\left(\frac{\omega}{k}\right)^2\left(\frac{1}{k}\right)^2} = \frac{Fk^4}{\rho\omega^2}$$

$\Pi_2 = aK \implies$ tells if waves are linear or not

$\Pi_3 = DK \implies St = \frac{\omega D}{\omega/k} = DK$

$\Pi_4 = KH \implies$ tells if deep or shallow

$\Pi_5 = \frac{\rho\left(\frac{\omega}{k}\right)\left(\frac{1}{k}\right)}{\mu} = \frac{\rho\omega}{k^2\mu} \implies$ Reynolds number

$\Pi_6 = \frac{g\left(\frac{1}{k}\right)}{(\omega/k)^2} = \frac{gk}{\omega^2} \implies \frac{1}{Fr}$ Froude number $\Longleftrightarrow$ dispersion relation

$Re = \frac{\rho UD}{\mu}$

$St = \frac{fD}{U}$

$C_f = \frac{F}{\frac{1}{2}\rho U^2 D^2}$

$F_r = \frac{U^2}{gL}$

$J = \frac{ND}{U}$

b) All π-groups in (a) must be same, except for π_5

d)

	D	H	ω	a	K	ak	KH	Dk	$\frac{gk}{\omega^2}$
full scale	4 m	100 m	$1 \frac{rad}{s}$	1 m	1 $k = \frac{\omega^2}{g} = \frac{1}{10}$	2 $\frac{1}{10}$	3 10	4 $\frac{4}{10}$	1
						↓	↓	↓	
model	$\frac{4}{5}$ m	6 20 m	$\sqrt{gk} = \sqrt{5}$	$\frac{\frac{1}{10}}{\frac{1}{2}} = \frac{1}{5}$	5 $k = \frac{Dk}{D} = \frac{1}{2}$	$\frac{1}{10}$	10	4/10	1

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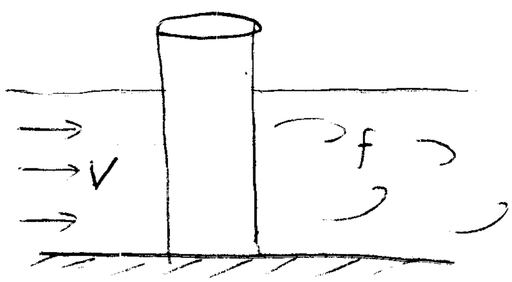
d) $Re = \dfrac{\text{inertial force}}{\text{viscous force}} = \dfrac{\rho\omega}{k^2\mu}$

e)

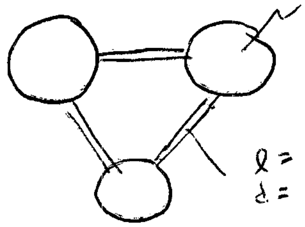


i) $St = \dfrac{fD}{V}$

ii) velocity not important ⇒ high Re ⇒ $St = 0.2$

$$f = \frac{0.2V}{D}$$

B2)



$L = 1\,m$
$D = 0.2\,m \rightarrow A = \frac{\pi(0.2)^2}{4} = 0.03\,m^2$
$C_D = 0.8$

$l = 0.2\,m$
$d = 0.05\,m$ } $A = (0.2)(0.05) = 0.01\,m^2$
$C_D = 1.2$

$$T = Drag = 3\left[\tfrac{1}{2}\rho U^2 C_D A\right] + 3\left[\tfrac{1}{2}\rho U^2 C_D A\right], \quad U = 1\,m/s$$

$$= 3\cdot\tfrac{1}{2}\cdot 1000\cdot 1^2\cdot 0.8\cdot 0.03 + 3\cdot\tfrac{1}{2}\cdot 1000\cdot 1^2\cdot 1.2\cdot 0.01$$

$$T = 54\,N$$

b) $P_{ower} = T\cdot V = 54\,W$

$$P_{wr} = \phi(d_{prop}, \rho, \mu, N, V)$$

$\frac{kg\,m^2}{s^3}$ m $\frac{kg}{m^3}$ $\frac{kg}{m s}$ $\frac{1}{s}$ $\frac{m}{s}$

$$\pi_1 = \frac{\rho V d}{\mu} = Re, \quad \pi_2 = \frac{Nd}{V} = J, \quad \pi_3 = \frac{P_{wr}}{\rho V^3 d^2}$$

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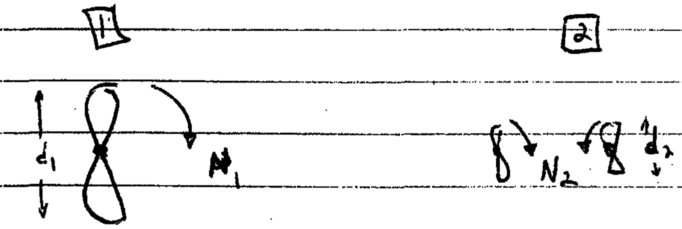
c)



$$N = \frac{JV}{D}$$

	Power	d	
$\boxed{1}$	54 N	$d = \left(\frac{P_{wr}}{\rho V^3 \pi}\right)^{1/2} \propto (P_{wr})^{1/2}$	
$\boxed{2}$	27 N	$d_2 = \frac{1}{\sqrt{2}} d_1$	$N_2 = \sqrt{2}\, N_1$
	half the power	smaller size	faster speed

B3)

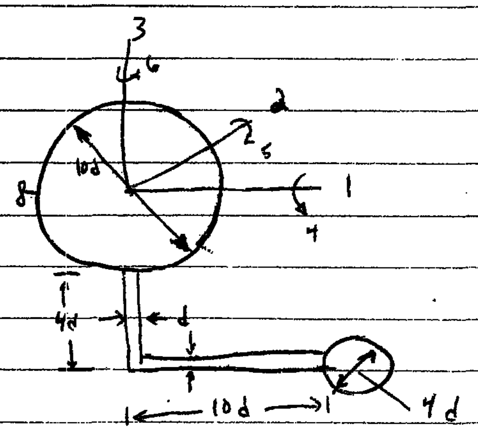
a)

	1	2	3	4	5	6
1	x	0	0	0	x	0
2		x	0	x	0	x
3			x	0	x	0
4	symmetric			x	0	x
5					x	0
6						x

$M_{a_{sphere}} = \rho \frac{2}{3} \pi R^3$

$M_{a_{cylinder}} = \rho \pi R^2 L$

$M^{(2D)}_{a_{circle}} = \rho \pi r^2$



$$M_{11} = \rho \frac{2}{3}\pi (5d)^3 + \rho \pi \left(\frac{d}{2}\right)^2 (4d) + \rho \frac{2}{3}\pi (2d)^3 = 90 \rho \pi d^3$$

$$M_{22} = \rho \frac{2}{3}\pi (5d)^3 + \rho \pi \left(\frac{d}{2}\right)^2 \left(4d + 10d\right) + \rho \frac{2}{3}\pi (2d)^3 = 95 \rho \pi d^3$$

$$M_{33} = \rho \frac{2}{3}\pi (5d)^3 + \rho \pi \left(\frac{d}{2}\right)^2 (10d) + \rho \frac{2}{3}\pi (2d)^3 = 94 \rho \pi d^3$$

$\vdots$

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B3 b)

$$F_1 = -\dot{U}_1 m_{11} - \dot{U}_5 m_{15}$$

$$m_{15} = \rho \tfrac{2}{3}\pi (2d)^3 \cdot 9d + \int_{5d}^{9d} \rho \pi \left(\tfrac{d}{2}\right)^2 z \, dz$$

$$= \rho\, 48 \pi d^4 + \rho \tfrac{1}{4}\pi d^2 \left(\tfrac{1}{2}\right)\left((9d)^2 - (5d)^2\right)$$

$$= 55 \rho \pi d^4$$

$$F_1 = -(1\,\text{m/s}^2)(90 \rho \pi d^3 \ \text{kg}) - (1\ \tfrac{\text{rad}}{\text{s}^2})(55 \rho \pi d^4 \ \text{kg m})$$

$$= -(90 + 55d)\, \rho \pi d^3$$

$$M_2 = -\dot{U}_1 m_{51} - \dot{U}_5 m_{55} + U_2 U_1 \cancel{m_{32}}^{\,0}$$

$$m_{55} = \int_{5d}^{9d} \rho \pi \left(\tfrac{d}{2}\right)^2 z^2 \, dz + \int_0^{10d} \rho \pi \left(\tfrac{d}{2}\right)^2 x^2 \, dx + \rho \tfrac{2}{3}\pi (2d)^3 \left((9d)^2 + (10d)^2\right)$$

$$= \rho \pi \left(\tfrac{d}{2}\right)^2 \tfrac{1}{3}\left((9d)^3 - (5d)^3\right) + \rho \pi \left(\tfrac{d}{2}\right)^2 \tfrac{1}{3} (10d)^3 + \rho \tfrac{2}{3}\pi (2d)^3 (181 d^2)$$

$$m_{55} = 1099 \rho \pi d^5$$

$$m_{32} = 0$$

$$M_2 = -(1\,\text{m/s}^2)(55 \rho \pi d^4) - (1\ \tfrac{\text{rad}}{\text{s}^2})(1099 \rho \pi d^5)$$

$$= -(55 + 1099 d)\, \rho \pi d^4$$