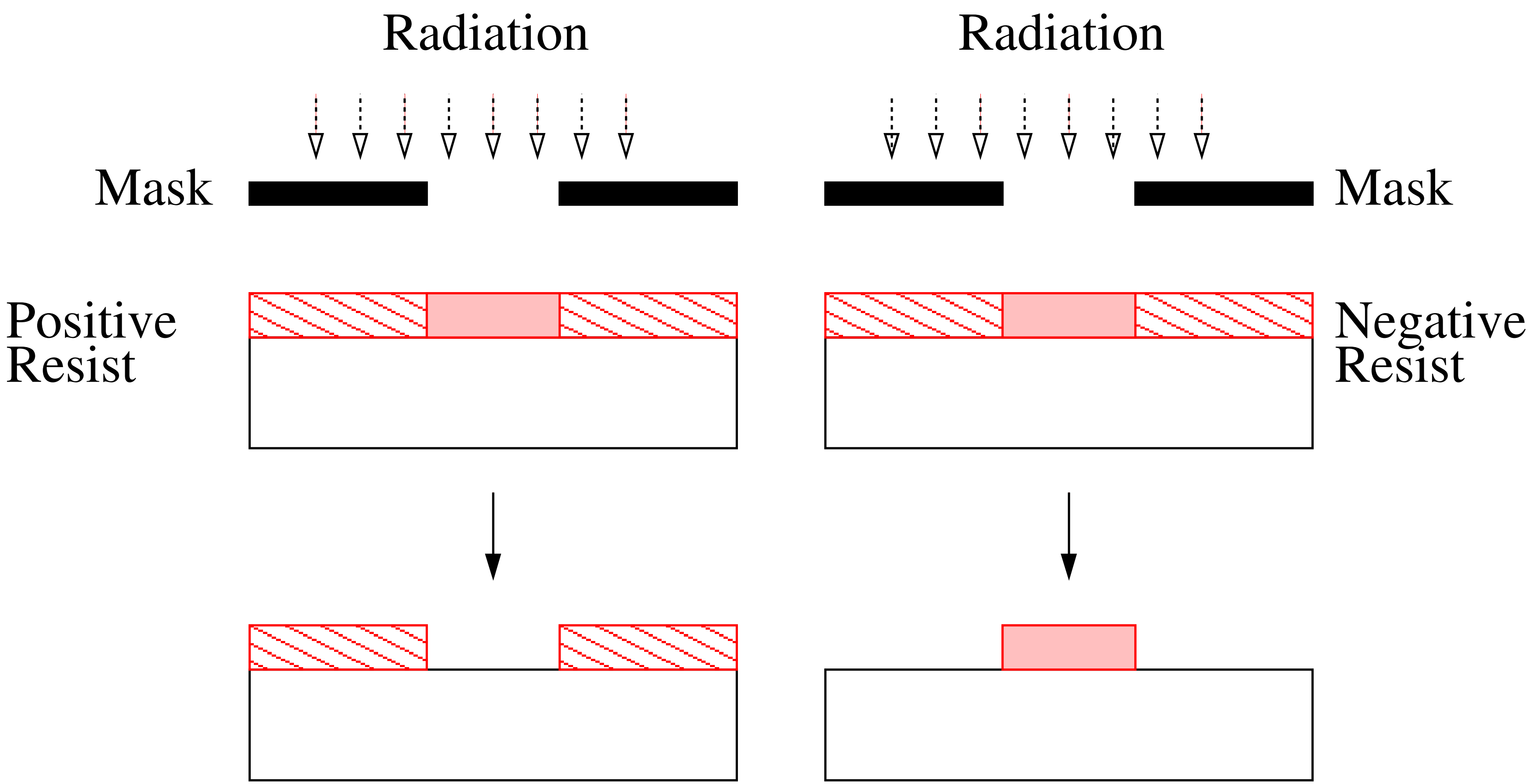
2.008 Quiz 2 Review

Quiz Topics

- **Casting:** V/SA vs. $(V/SA)^2$
- **Forming:** Force, elastic v. plastic deformation.
- **Process:** shop,project,flow,cell.
- **Systems:** $L = \lambda w$
- **Quality:** C_{pk}, SPC.
- **Cost:** allocations
- **Layered Manufacturing**
- **MEMS** Look this over!

Photo Resists



Laser Cutting Problem

A laser cutter cuts its material by evaporating the material along the cut line. In terms of the following variables:

Quantity	Variable	Units
Mass	m	grams
Ambient Temperature	t_o	°K
Melting Temperature	t_m	°K
Boiling Temperature	t_b	°K
Solid specific heat capacity	c_p	$J(kG)^{-1} \, {}^{\circ}K^{-1}$
Latent heat of fusion	h_f	$J(kG)^{-1}$
Liquid specific heat capacity	c_l	$J(kG)^{-1} \, {}^{\circ}K^{-1}$
Latent heat of vaporization	h_v	$J(kG)^{-1} \, {}^{\circ}K^{-1}$
Density	ρ	kg/m^3
Material thickness	h	m
Radius of laser spot	r	m
Power of the laser	P	W

write a formula which describes how much energy per gram is required to vaporize a material.

Laser Cutting Problem

A laser cutter cuts its material by evaporating the material along the cut line. In terms of the following variables:

Quantity	Variable	Units
Mass	m	grams
Ambient Temperature	t_o	°K
Melting Temperature	t_m	°K
Boiling Temperature	t_b	°K
Solid specific heat capacity	c_p	$J(kG)^{-1} \circ K^{-1}$
Latent heat of fusion	h_f	$J(kG)^{-1}$
Liquid specific heat capacity	c_l	$J(kG)^{-1} \circ K^{-1}$
Latent heat of vaporization	h_v	$J(kG)^{-1} \circ K^{-1}$
Density	ρ	kg/m^3
Material thickness	h	m
Radius of laser spot	r	m
Power of the laser	P	W

write a formula which describes how much energy per gram is required to vaporize a material.

$$E_{vaporize} = m((t_m - t_o)c_p + h_f + (t_b - t_m)c_l + h_v)$$

Laser Cutting

The laser beam is focused onto a circle of radius r. Assume that all of the energy of the laser is all delivered to this circle, and that all of the energy is used in the cutting process (ie, there is no energy loss due to spectral absorption). Let ρ be the density of the material, and let h be the material thickness. Write an equation governing speed of cutting to the power of the laser, P.

Laser Cutting

The laser beam is focused onto a circle of radius r. Assume that all of the energy of the laser is all delivered to this circle, and that all of the energy is used in the cutting process (ie, there is no energy loss due to spectral absorption). Let ρ be the density of the material, and let h be the material thickness. Write an equation governing speed of cutting to the power of the laser, P.

The laser's energy, P, is delivered uniformly over an area of πr^2. In order to cut through this material, a volume of $\pi r^2 h$ mm^3 must be evaporated. This volume will require

$$e = \frac{\pi r^2 h E_{vaporize}}{\rho} \text{ Joules}$$

of energy to vaporize. If P represents watts of laser power available, then P/e seconds are required for the laser to cut through the material. Since the laser can safely move $2r$ during this interval, the cutting velocity can be

$$v_c = 2re/P$$

This assumes that none of the heat is lost to diffusion. In fact, if v_c is greater than the rate of diffusion, this is assumption is valid.

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning
- Sand casting
- Injection molding
- Milling
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting
- Injection molding
- Milling
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding
- Milling
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling Delivery of power to cutting tool
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling Delivery of power to cutting tool
- Forging Force required to plastically deform metal
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling Delivery of power to cutting tool
- Forging Force required to plastically deform metal
- Die casting Conductance of heat
- Thermoforming
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling Delivery of power to cutting tool
- Forging Force required to plastically deform metal
- Die casting Conductance of heat
- Thermoforming Heating of material to glassy state
- MIG Welding
- Friction Welding

Limiting Factors

What *physical* quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
- Sand casting Diffusion of heat
- Injection molding Diffusion of heat
- Milling Delivery of power to cutting tool
- Forging Force required to plastically deform metal
- Die casting Conductance of heat
- Thermoforming Heating of material to glassy state
- MIG Welding Delivery of current to melt weld area
- Friction Welding Generating enough heat via friction

Short Answers

- Why are nucleation agents added to a metal casting?

Short Answers

- Why are nucleation agents added to a metal casting?

  Nucleation agents promote equaxis crystal growth.
- What happens to grain size as the cooling time increases?

Short Answers

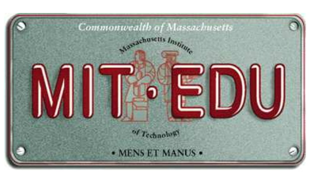
- Why are nucleation agents added to a metal casting?

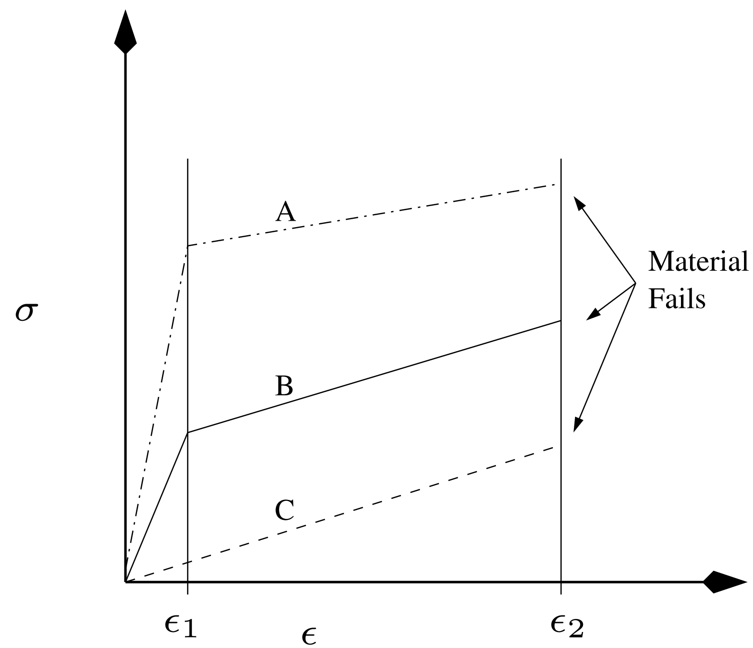
  Nucleation agents promote equaxis crystal growth.
- What happens to grain size as the cooling time increases?

  Grain size decreases as cooling time increases.

Forming [Q2 '02]

You have been assigned to set up a manufacturing process for making license plates. You will be responsible for setting up the line and ensuring it meets rate, quality and cost goals.

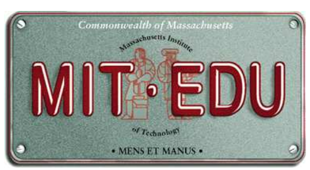


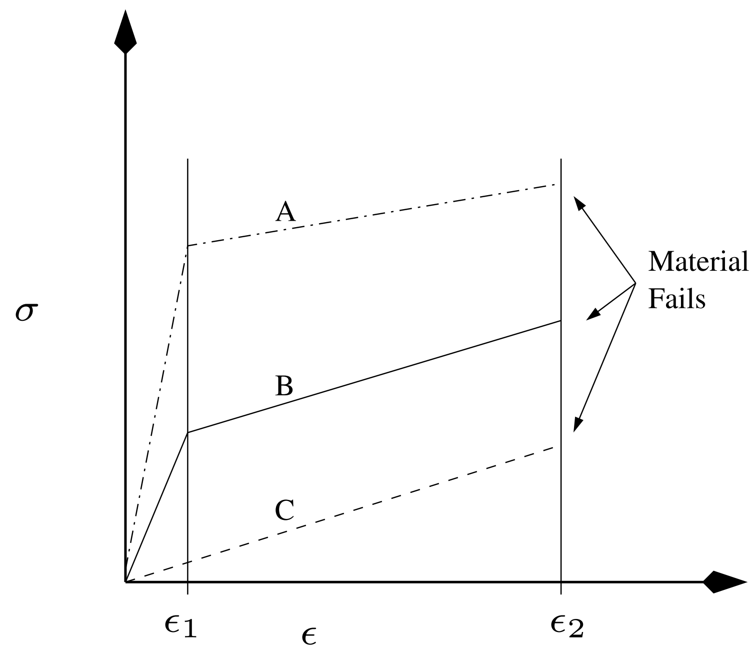


Which (if any) of these materials (A and/or B and/or C) can not be used to form the license plate? Justify your answer with a short explanation.

Forming [Q2 '02]

You have been assigned to set up a manufacturing process for making license plates. You will be responsible for setting up the line and ensuring it meets rate, quality and cost goals.





Which (if any) of these materials (A and/or B and/or C) can not be used to form the license plate? Justify your answer with a short explanation.

Solution: Material C is not usable because it has no plastic deformation before failing – it is perfectly elastic.

Forming [Q2 '02]

In the following process:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
1		2		3		4		5		6

the maximum strain (ϵ_m) in the license plate falls within one of three categories:

a: $0 < \epsilon_m < \epsilon_1$ b:$\epsilon_1 < \epsilon_m < \epsilon_2$ c: $\epsilon_m = \epsilon_2$

Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

Operation 1	Operation 2	Operation 3	Operation 4

Forming [Q2 '02]

In the following process:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
1		2		3		4		5		6

the maximum strain (ϵ_m) in the license plate falls within one of three categories:

a: $0 < \epsilon_m < \epsilon_1$ b:$\epsilon_1 < \epsilon_m < \epsilon_2$ c: $\epsilon_m = \epsilon_2$

Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

Operation 1	Operation 2	Operation 3	Operation 4
C			

Reasons

- Material is cut.

Forming [Q2 '02]

In the following process:

1	2	3	4	5	6
Blank plate shape →	Form outer ring →	Form letter + numbers →	Punch four holes →	Coat Paint Detail →	Inspect

the maximum strain (ϵ_m) in the license plate falls within one of three categories:

a: $0 < \epsilon_m < \epsilon_1$ b:$\epsilon_1 < \epsilon_m < \epsilon_2$ c: $\epsilon_m = \epsilon_2$

Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

Operation 1	Operation 2	Operation 3	Operation 4
C	B		

Reasons

- Material is cut.
- Material is plastically deformed to accept geometry.

Forming [Q2 '02]

In the following process:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
1		2		3		4		5		6

the maximum strain (ϵ_m) in the license plate falls within one of three categories:

a: $0 < \epsilon_m < \epsilon_1$ b:$\epsilon_1 < \epsilon_m < \epsilon_2$ c: $\epsilon_m = \epsilon_2$

Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

Operation 1	Operation 2	Operation 3	Operation 4
C	B	B	

Reasons

- Material is cut.
- Material is plastically deformed to accept geometry.
- Material is plastically deformed to accept geometry.

Forming [Q2 '02]

In the following process:

Blank plate shape →	Form outer ring →	Form letter + numbers →	Punch four holes →	Coat Paint Detail →	Inspect
1	2	3	4	5	6

the maximum strain (ϵ_m) in the license plate falls within one of three categories:

$$\text{a: } 0 < \epsilon_m < \epsilon_1 \qquad \text{b:} \epsilon_1 < \epsilon_m < \epsilon_2 \qquad \text{c: } \epsilon_m = \epsilon_2$$

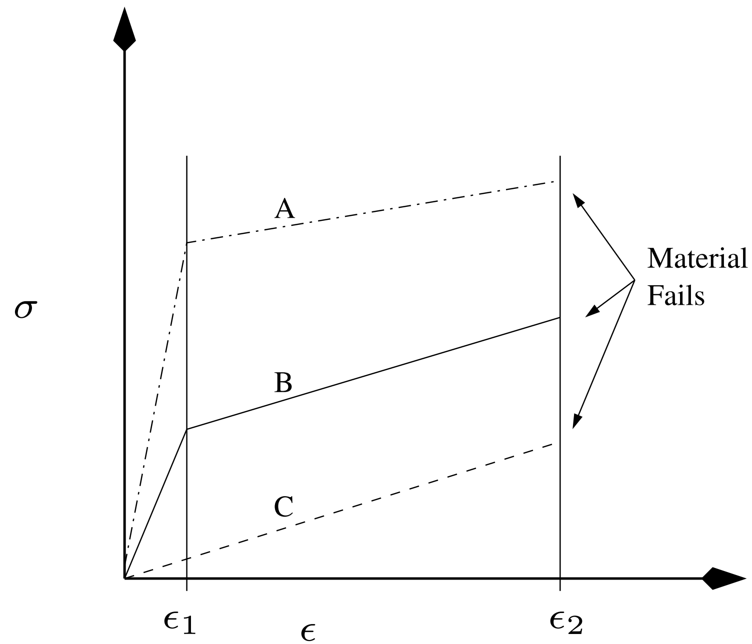
Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

Operation 1	Operation 2	Operation 3	Operation 4
C	B	B	C

Reasons

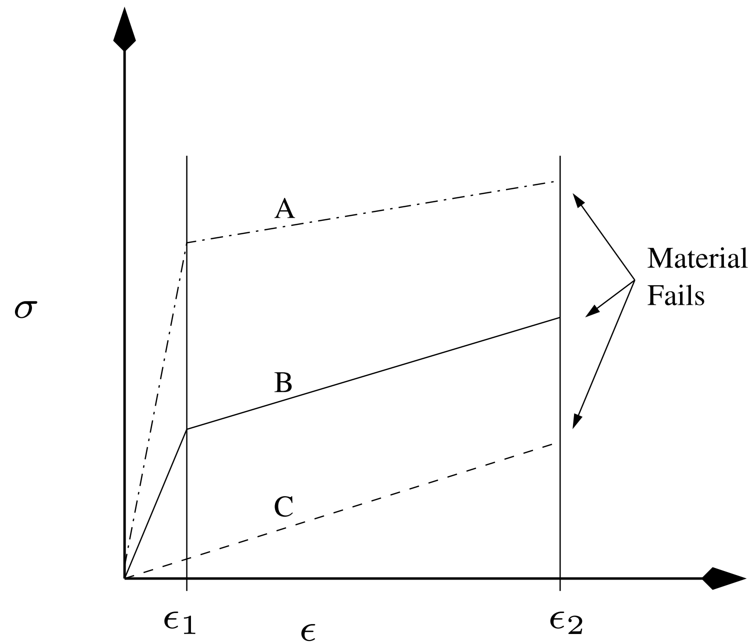
- Material is cut.
- Material is plastically deformed to accept geometry.
- Material is plastically deformed to accept geometry.
- Material is cut.

Forming [Q2 '02]



Given a constant stamping speed (speed of the die as it closes) and a specific license plate geometry, which of the usable materials (among A, B, or C) would you choose if you wanted to minimize the power/energy required to make the license plate? Justify your answer using words and basic physics/equations.

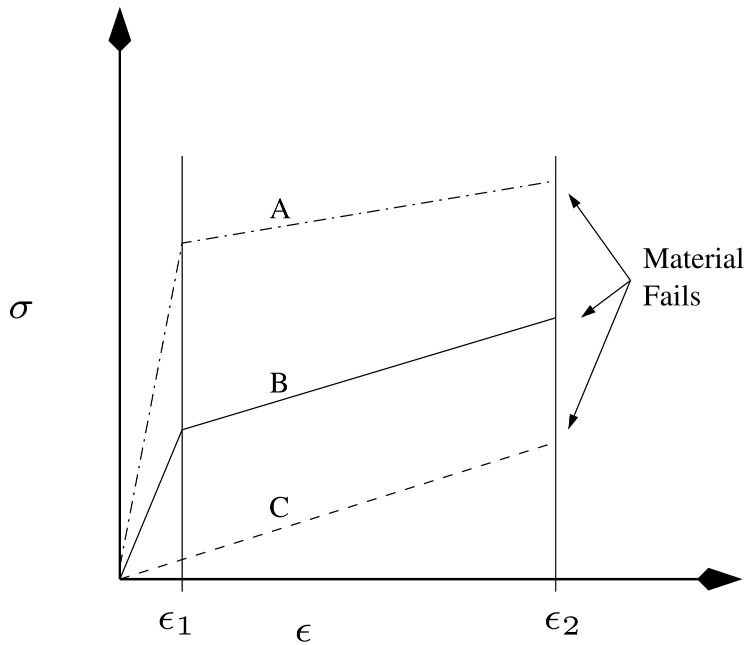
Forming [Q2 '02]



Given a constant stamping speed (speed of the die as it closes) and a specific license plate geometry, which of the usable materials (among A, B, or C) would you choose if you wanted to minimize the power/energy required to make the license plate? Justify your answer using words and basic physics/equations.

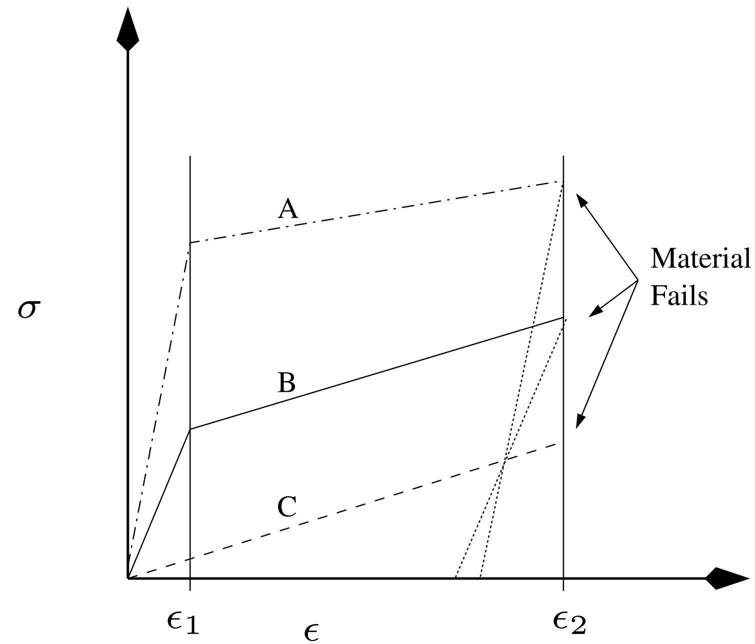
Material B minimizes the power/energy requirements. For a constant speed, the power is proportional to the energy, and energy is proportional to $\sigma(\epsilon^2)$, and so the lower stress curve represents lower required energy.

Forming [Q2 '02]



Which of the usable materials (among A, B, or C) would you choose to minimize the amount of spring back? Justify your answer.

Forming [Q2 '02]

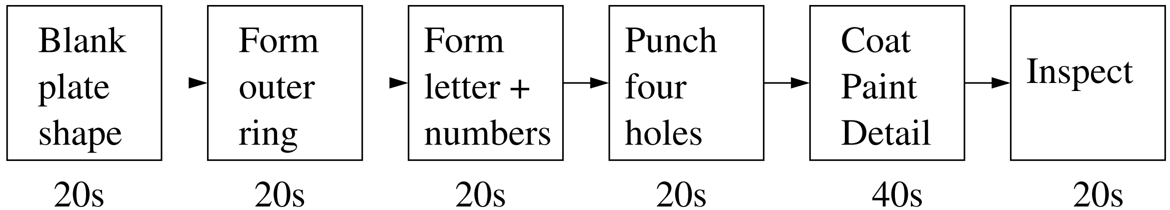


Which of the usable materials (among A, B, or C) would you choose to minimize the amount of spring back? Justify your answer.

Material A minimizes spring back; following the path of elastic modulus back from the ultimate forming strain results in material B recovering more than material A, as shown below.

Forming [Q2 ’02]

Given the following processing times (per part) for the manufacturing line:



calculate the production rate in parts/hour.

Forming [Q2 '02]

Given the following processing times (per part) for the manufacturing line:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
20s		20s		20s		20s		40s		20s

calculate the production rate in parts/hour.

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
20s		20s		20s		20s		40s		20s
180p/h		180p/h		180p/h		180p/h		90p/h		180p/h

The painting step limits the process:

$$60s/m/40s/part * 60m/hr = 90parts/h$$

Forming [Q2 '02]

Now consider the following MTTF/MTTR data for each machine:

Blank plate shape → Form outer ring → Form letter + numbers → Punch four holes → Coat Paint Detail → Inspect

Blank plate shape	Form outer ring	Form letter + numbers	Punch four holes	Coat Paint Detail	Inspect
MTTF=4.5h	MTTF=3h	MTTF=3h	MTTF=40m	MTTF=45m	MTTF=4.75h
MTTR=30m	MTTR=20m	MTTR=20m	MTTR=60m	MTTR=5m	MTTR=15m

Calculate the upper bound on production rate in parts/hour.

Forming [Q2 ’02]

Now consider the following MTTF/MTTR data for each machine:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
MTTF=4.5h		MTTF=3h		MTTF=3h		MTTF=40m		MTTF=45m		MTTF=4.75h
MTTR=30m		MTTR=20m		MTTR=20m		MTTR=60m		MTTR=5m		MTTR=15m
0.90		0.90		0.90		0.40		0.90		0.95

Calculate the upper bound on production rate in parts/hour.

Compute MTTF/(MTTF+MTTR) for each process. Then multiply this availability factor by each of the production rates. The punch operation has $\frac{40}{40+60} = 0.40$.

$$Rate_{punch} = (0.40)(60\text{sec/min}/20\text{sec/part} * 60\text{min/hr}) = 72 \text{ parts/min}$$

Forming [Q2 '02]

Now consider the following MTTF/MTTR data for each machine:

Blank plate shape	→	Form outer ring	→	Form letter + numbers	→	Punch four holes	→	Coat Paint Detail	→	Inspect
MTTF=4.5h		MTTF=3h		MTTF=3h		MTTF=40m		MTTF=45m		MTTF=4.75h
MTTR=30m		MTTR=20m		MTTR=20m		MTTR=60m		MTTR=5m		MTTR=15m
0.90		0.90		0.90		0.40		0.90		0.95

Calculate the upper bound on production rate in parts/hour.

Compute MTTF/(MTTF+MTTR) for each process. Then multiply this availability factor by each of the production rates. The punch operation has $\frac{40}{40+60} = 0.40$.

$$Rate_{punch} = (0.40)(60\text{sec/min}/20\text{sec/part} * 60\text{min/hr}) = 72 \text{ parts/min}$$

Would adding a single buffer help? If so, where?

Forming [Q2 '02]

Now consider the following MTTF/MTTR data for each machine:

Blank plate shape →	Form outer ring →	Form letter + numbers →	Punch four holes →	Coat Paint Detail →	Inspect
MTTF=4.5h	MTTF=3h	MTTF=3h	MTTF=40m	MTTF=45m	MTTF=4.75h
MTTR=30m	MTTR=20m	MTTR=20m	MTTR=60m	MTTR=5m	MTTR=15m
0.90	0.90	0.90	0.40	0.90	0.95

Calculate the upper bound on production rate in parts/hour.

Compute MTTF/(MTTF+MTTR) for each process. Then multiply this availability factor by each of the production rates. The punch operation has $\frac{40}{40+60} = 0.40$.

$$Rate_{punch} = (0.40)(60\text{sec/min}/20\text{sec/part} * 60\text{min/hr}) = 72 \text{ parts/min}$$

Would adding a single buffer help? If so, where?

Add a buffer after the punch to offset machine downtime.

Forming [Q2 '02]

Another option for increasing the production rate is adding a machine. Option 1 is to rent a manual machine, which will require an additional person to run; Option 2 is purchasing a more expensive automatic machine that can be run by the existing operator.

Profit:	$1.50/part
Shift time:	2000 hours/year
Labor rate:	$20/hour
Labor overhead:	20 %
Manual machine:	$3,000/month
Automatic machine	$100,000

Assuming you want the decision to produce the lowest cost after one year, which option is best?

Forming [Q2 ’02]

Another option for increasing the production rate is adding a machine. Option 1 is to rent a manual machine, which will require an additional person to run; Option 2 is purchasing a more expensive automatic machine that can be run by the existing operator.

Profit:	\$1.50/part
Shift time:	2000 hours/year
Labor rate:	\$20/hour
Labor overhead:	20 %
Manual machine:	\$3,000/month
Automatic machine	\$100,000

Assuming you want the decision to produce the lowest cost after one year, which option is best?

Option 1 cost = ([3000 \$/mo] + [20 \$/hr] * [1.2] * [167 hrs/mo]) * [12 months] = \$ 84096

Option 2 cost = \$100,000

Forming [Q2 '02]

Another option for increasing the production rate is adding a machine. Option 1 is to rent a manual machine, which will require an additional person to run; Option 2 is purchasing a more expensive automatic machine that can be run by the existing operator.

Profit:	$1.50/part
Shift time:	2000 hours/year
Labor rate:	$20/hour
Labor overhead:	20 %
Manual machine:	$3,000/month
Automatic machine	$100,000

Assuming you want the decision to produce the lowest cost after one year, which option is best?

Option 1 cost = ([3000 $/mo] + [20 $/hr] * [1.2] * [167 hrs/mo]) * [12 months] = $ 84096
Option 2 cost = $100,000
How much time is required before the options are equal from a financial perspective?

Forming [Q2 '02]

Another option for increasing the production rate is adding a machine. Option 1 is to rent a manual machine, which will require an additional person to run; Option 2 is purchasing a more expensive automatic machine that can be run by the existing operator.

Profit:	$1.50/part
Shift time:	2000 hours/year
Labor rate:	$20/hour
Labor overhead:	20 %
Manual machine:	$3,000/month
Automatic machine	$100,000

Assuming you want the decision to produce the lowest cost after one year, which option is best?

Option 1 cost = ([3000 $/mo] + [20 $/hr] * [1.2] * [167 hrs/mo]) * [12 months] = $ 84096
Option 2 cost = $100,000
How much time is required before the options are equal from a financial perspective?
([3000 $/mo] + [20 $/hr] * [1.2] * [167 hrs/mo]) * [m months] = $ 100,000 $\rightarrow m = 14.3$ mo

Forming [Q2 ’02]

In addition to rate, quality is an issue. Assuming there are two painting stations, the two stations do not provide the same quality. The main factor for quality is alignment of the paint layer with the embossed features in the sheet metal.
Both the average value for alignment and distribution of the alignment are important to overall quality. The two stations in the manufacturing line differ in both mean value and standard deviation.

Machine A: $\overline{x} = 0.80$mm, $\sigma = 0.15$ mm

Machine B: $\overline{x} = 0.00$ mm, $\sigma = 0.48$ mm

The specifications for painting the license plates call for alignment of 0 ± 1.0mm.
Calculate C_p and C_{pk} for each machine.

Forming [Q2 '02]

In addition to rate, quality is an issue. Assuming there are two painting stations, the two stations do not provide the same quality. The main factor for quality is alignment of the paint layer with the embossed features in the sheet metal.
Both the average value for alignment and distribution of the alignment are important to overall quality. The two stations in the manufacturing line differ in both mean value and standard deviation.

Machine A: $\overline{x} = 0.80$mm, $\sigma = 0.15$ mm
Machine B: $\overline{x} = 0.00$ mm, $\sigma = 0.48$ mm

The specifications for painting the license plates call for alignment of 0 ± 1.0mm.
Calculate C_p and C_{pk} for each machine.

A: $C_p =$ Range$/(6\sigma) = [2mm]/[0.9mm] = 2.22$
B: $C_p = [2mm]/[2.88mm] = 0.69$

Forming [Q2 '02]

Calculate the resulting yield (percentage of parts within the specifications) for each machine.

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611

Forming [Q2 '02]

Calculate the resulting yield (percentage of parts within the specifications) for each machine.

Use the charts for Z :

A

$$
\begin{aligned}
Z_{\text{min}} &= \frac{1-0.8}{0.15} = 1.33 \rightarrow [P = 0.9082], (1-P) = 0.0918 \\
Z_{\text{max}} &= \frac{1+0.8}{0.15} = 12.0 \rightarrow [P \approx 1.000], (1-P) = 0 \\
\text{Yield} &= 1 - 0.0918 - 0.0000 = 0.9082 = 90.8
\end{aligned}
$$

B

$$
\begin{aligned}
Z_{\text{min}} &= \frac{1-0}{0.48} = 2.08 \rightarrow P = 0.9812, (1-P) = 0.0188 \\
Z_{\text{max}} &= \frac{1-0}{0.48} = 2.08 \rightarrow P = 0.9812, (1-P) = 0.0188 \\
\text{Yield} &= 1 - 0.0188 - 0.0188 = 0.9624 = 96.2
\end{aligned}
$$

Homework 7-3

Your enterprise is using a process which produces parts whose length have a mean 2.5in and standard deviation 0.015in. You plan to make 100,000 of these parts. You have determined by taking into account the rework cost and the loss of customer satisfaction that the quality loss function for the process is:

$$L = 2250\sigma^2$$

where σ is the standard deviation of length of the part and L is the average quality lost in dollars per part.
A very smart but somewhat expensive Harvard graduate has offered you her expert advice on your process. For every 0.001in reduction in the standard deviation of the length of the part, she will charge $\$6000$.
How much of her advice do you buy? (You can buy fractional amounts of her consulting).

Homework 7-3

Your enterprise is using a process which produces parts whose length have a mean 2.5in and standard deviation 0.015in. You plan to make 100,000 of these parts. You have determined by taking into account the rework cost and the loss of customer satisfaction that the quality loss function for the process is:

$$L = 2250\sigma^2$$

where σ is the standard deviation of length of the part and L is the average quality lost in dollars per part.

A very smart but somewhat expensive Harvard graduate has offered you her expert advice on your process. For every 0.001in reduction in the standard deviation of the length of the part, she will charge \$$6000$.

How much of her advice do you buy? (You can buy fractional amounts of her consulting).

Let $C(x)$ be the cost function. Minimize by taking derivatives:

$$\begin{aligned} C(x) &= (10^5)2250(0.015 - 0.001x)^2 + 6000x \\ &= 225x^2 - 750x + 50625 \\ C'(x) &= 450x - 750 \\ x &= 1.66 \end{aligned}$$

It pays to buy 1.66 units of consulting.