

## 2.004: MODELING, DYNAMICS, & CONTROL II

Fall Term 2002

PLEASE ALSO NOTE THAT ALL PRELAB EXERCISE ARE DUE AT THE START (WITHIN 10 MINUTE) OF THE LAB SESSION, NO LATE WORK IS ACCEPTED.

### Pre-Lab Exercise for Experiment 3

We will consider the dynamics of a bouncing ball, with mass  $m$  under gravity  $g$ .

(1) Quantitatively describe the trajectory  $z(t)$  of a ball in between bounces. You may assume the maximum height that it reaches is  $h$ . What is the functional form of the trajectory?

$$z(t) = \frac{1}{2} g t^2 + v_0 t + z_0 \quad \text{where } g = -9.8 \text{ m/s}^2, \quad \uparrow +$$

The trajectory followed is a parabola whose properties are defined by the initial position and velocity, as well as the influence of gravity. Given that the maximum height that the ball reaches is  $h$ , we can use conservation of energy to determine the value of  $v_0$ .

$$\begin{aligned} T_i + P_i &= T_f + P_f \\ \frac{1}{2} m v_0^2 + 0 &= 0 + mgh \\ v_0 &= \sqrt{2gh} \end{aligned}$$

Therefore

$$z(t) = \frac{1}{2} g t^2 + \sqrt{2gh} t$$

(2) Quantitatively describe the time dependence of the velocity,  $v(t)$ , and  $a(t)$  of the ball using result from (1).

$$\begin{aligned} v(t) &= d/dt(z(t)) \\ &= g t + \sqrt{2gh} \\ a(t) &= d/dt(v(t)) = d^2/dt^2(z(t)) \\ &= g \end{aligned}$$

again,  $g$  is defined as  $-9.8 \text{ m/s}^2$

(3) Consider a collision event, assume the coefficient of restitution is  $e$  and the ball reaches a maximum height,  $h$ , before impact. Calculate  $v(t)$  and  $a(t)$  before and after impact.

Before impact:

↑+

$$v_i = \sqrt{2gh}$$

$$a_i = g$$

After impact:

$$v_f = ev_i = -e\sqrt{2gh}$$

$$a_f = g$$

(4) What is the momentum transfer resulted from the impact?

$$\begin{aligned}\Delta p &= m(v_f - v_i) \\ &= m(-e\sqrt{2gh} - \sqrt{2gh}) \\ &= -m\sqrt{2gh}(e+1)\end{aligned}$$

↑+

(5) Can you define impulse and force during impact? What information do you need to deduce the force generated at impact?

Impulse is the change in momentum of the ball, as solved for above in question 4.

Force is the change in momentum divided by the time of contact in which the momentum changed. For these impacts, the duration of contact is far too short for us to measure accurately on our stopwatches.

(6) You model the trajectory and the kinematic parameters of the ball as piece-wise continuous functions. However, since the camera that you will be using to capture the data has finite frame rate, you get data at discrete time points. Your data looks like:

$z(t_0), z(t_1), z(t_2), \dots, z(t_n), \dots, z(t_{M-1}), z(t_M)$ , where  $t_{n+1} - t_n = \Delta t$  is a constant.

How do you calculate  $v(t_n)$ ,  $a(t_n)$ ,  $p(t_n)$ , and  $f(t_n)$ ?

using a for loop to iterate through the length of your  $z$  vector (for  $n=1:M-1$ )

$$v(t_n) = [z(t_{n+1}) - z(t_n)] / \Delta t$$

$$a(t_n) = [v(t_{n+1}) - v(t_n)] / \Delta t$$

$$p(t_n) = m[v(t_{n+1}) - v(t_n)]$$

$$f(t_n) = m[v(t_{n+1}) - v(t_n)] / \Delta t$$