2.004 Dynamics and Control II
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

2.004 *Dynamics and Control II*

Laboratory Session 3:
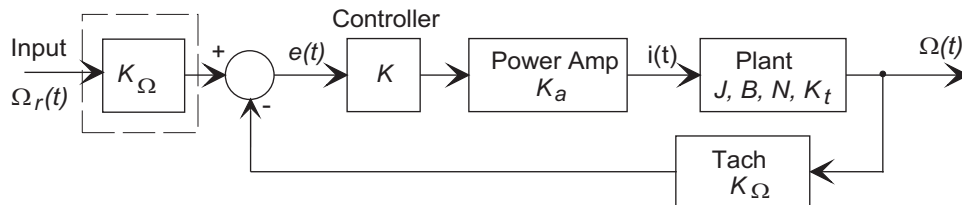# Construction of a Proportional Velocity Controller[1]

Laboratory Objectives:

(i) Construction and testing of an op-amp differential amplifier.

(ii) Use of the differential amplifier as a proportional velocity controller for the 2.004 rotational plant.

(iii) Measurement of the transient response characteristics, and the steady-state behavior of the closed-loop controller.

(iv) Comparison of the measured performance with predictions based on the pre-lab exercise.

Reading:
Read the course handout *Introduction to the Operational Amplifier*, especially the description of the *differential amplifier* (page 7).

Lab Preparation Exercise: The following was set as Problem 1 in Problem Set 4. You should do this problem before the lab, and bring your results with you.

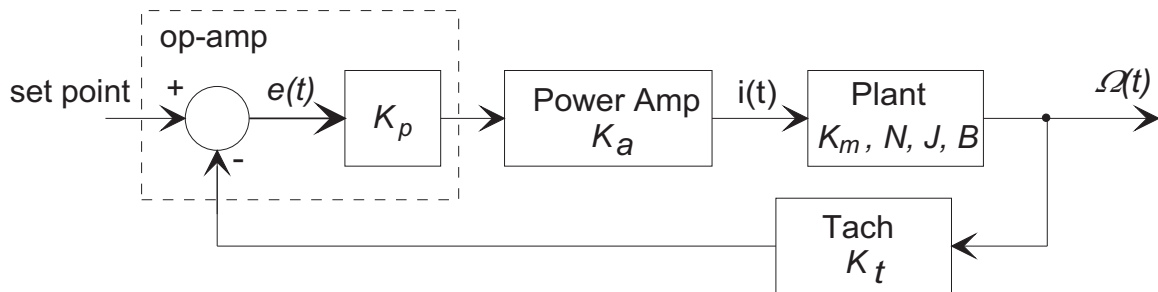The closed-loop control system for this lab is shown above.

(a) Derive the transfer function relating the angular velocity Ω of the motor to the voltage input command in terms of the controller gain K, servo amp gain K_a, motor torque constant K_t, and tachometer gain K_Ω. Ignore nonlinearities such as Coulomb friction. (Question: Why did I include the block K_Ω, in the input?)

(b) Now put in numerical values:

- You computed the value of J.
- The motor-drive gears have 44 and 180 teeth.
- The value of $K_a = 2$ amps/volt. The value of $K_\Omega = 0.016$ v/rpm.
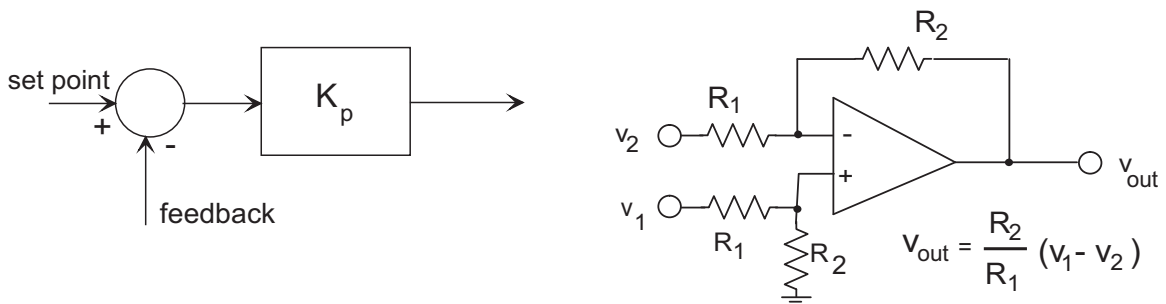
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[1]February 29, 2008

- Assume a value $K = 3$ for the controller.

- We will start with one damping magnet. From Lab. 1, an average over several groups for the damping with one magnet was $B = 0.014$ N.m.s/rad.

(b) With all of the numerical values substituted, except for the controller gain K, derive the closed loop time-constant in terms of K.

(c) Similarly, determine the steady-state error to a constant (step) input as a function of the controller gain K.

Hint: You may find certain similarities between this problem and the automobile cruise control example we discussed in class.
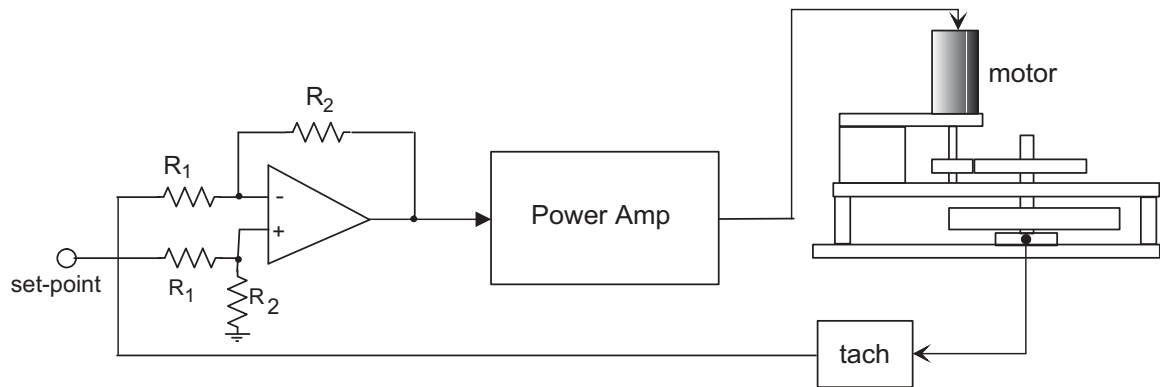
Introduction In this lab session you will construct and test a closed-loop velocity control scheme as shown in the block diagram below:

We will use a differential amplifier, built from a single I.C. op-amp to implement the summer and gain blocks in the controller.

The differential amplifier operation is described in the class handout *Introduction to the Operational Amplifier*, and it is assumed that you have read and understand this handout. The overall closed-loop connections will be as shown below:
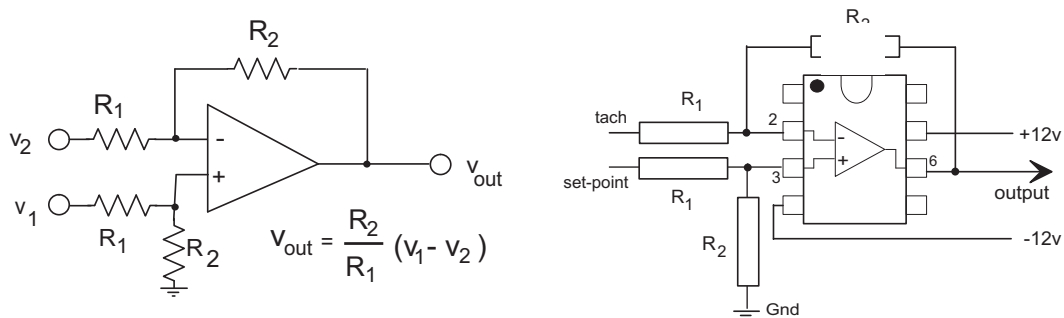
The differential amplifier uses four resistors, two with value R_1 ohms, and two with value R_2 ohms. The amplifier output voltage V_o is

$$v_o = \frac{R_2}{R_1}(v_1 - v_2)$$

where we choose R_1 and R_2 so that $R_2/R_1 = K_p$. In this lab, we will use the value $R_1 = 10k$ (10,000) ohms, and choose the value of R_2 to set the proportional gain $K_p = 3$. (This lab session will use just a single controller gain. In Lab 4 you will make a detailed study of the response at various controller gains.)

The circuit will be built on a proto-board (breadboard). Your instructor will explain the structure of the board to you and show you how to connect the power supply. You will be supplied with all of the components and tools necessary to build the circuit.

Construction of an op-amp summing amplifier: Refer to the class handout *Introduction to the Operational-Amplifier* (pages 6 and 7) for a description of a differential amplifier. Construct the amplifier in the lower left-hand corner of your breadboard and use the first two (left) BNC connectors as inputs (set-point and tachometer feedback), and the third BNC connector as the output that will drive the power amplifier. (Do not connect the op-amp to the motor and power amplifier yet.) The circuit and a suggested layout are shown below:

(1) Make sure the power supply is disconnected.

(2) Place the op-amp over the central rail near the lower left-hand corner of the circuit. Leave some space below it, but make sure that its pins do not line up with gaps on the side rails that deliver power and ground.

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(3) Connect $+V_{ss}$ (pin 7) to the +12 V input rail and $-V_{ss}$ (pin 4) to the -12 V input rail.

(4) Install the feedback resistor R_2 across the top of the op-amp. Use two jumpers that differ in length by 0.1" to connect the inverting input (pin 2) and the output (pin 6) to the same row above the op-amp.

(5) Using vertical and horizontal wiring, connect each input (i.e., the first two BNC connectors) via a resistor of value $R_1 = 10k\Omega$ to the two inputs (pins 2 and 3) as in the circuit diagram.

(6) Connect a resistor of value R_2 from the non-inverting input to ground (0 v.).

(7) Connect the output (pin 6) to the third BNC connector.

(8) Check your wiring for mistakes. When you think everything is correct, turn on the power.

(9) Use the function generator to supply an input voltage waveform to one of the inputs (while grounding the other) and monitor the output on the Chart Recorder. Are the input and output in phase? Now apply the function generator signal to both inputs. Does the differential amplifier live up to its name?

Wiring Guidelines: When wiring circuits on the circuit board, follow these guidelines:

- Make sure the power to the breadboard is disconnected. When the power is on, the red light-emitting diode (LED) on the breadboard is lit.

- When building circuits, lay down wires, resistors, and capacitors at right angles and keep them flush to the surface, if possible.

- It is important that you follow our suggested component layouts. (The instructors must be able to quickly spot wiring errors in order to help you debug the circuits) For example, the op-amp should be placed with pin 1 toward the top of the breadboard. Similarly, the op-amp feedback loop should consist of two wires going vertically from pins 2 and 6 to the same row in the circuit board above the op-amp, and should be closed by a resistor

- Use the supplied pre-cut wire jumpers where possible. Note that the length of the wires is specified using the standard color code (in increments of 0.1, and 1 inch): for example a 0.3" or 3" jumper is colored orange.

Observe the Closed-loop System Behavior: When circuit has been successfully constructed and debugged, you should examine the closed-loop system's behavior; in particular the response of the system to changes in the viscous friction coefficient B.

You are going to

- Subject the system to a step input in reference angular velocity, $r(t)$, and then

- Observe the system's

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- input,

- output (tachometer voltage), and

- controller output (which is proportional to the error)

with no viscous friction (magnets), then using one magnet (to represent a change in the system parameters.)

Procedure:

(a) Make sure that the power amplifier is turned OFF.

(b) Connect the BNC cables necessary to capture voltages representing the command (function generator output), tachometer output, and controller output, to channels 0, 1 and 2 of the Chart Recorder program.

(c) Set the function generator to output a *square wave* with 1.0v amplitude, 0.5 volt offset and 0.2 Hz frequency. (Note that the default frequency range for this signal is kHz.) Also make sure that the 50Ω terminator resistor is connected. *Do not* turn on the output from the function generator yet.

(d) Remove all magnets from under the copper disc.

(e) You should always take care when you first turn on a system with feedback. If you have mistakenly reversed some of the wires, the system can go unstable. Have an instructor check your configuration before turning on the power.

Note that there are two switches on the power amplifier - the main power switch, and a thermal circuit breaker switch in the center of the panel. First, turn the power amplifier's switch to the ON position, leaving the circuit breaker switch OFF. When you are ready to begin, turn the circuit breaker ON, but be ready to turn it off in a hurry if the motor races.

(f) If all seems well behaved, turn on the output from the function generator.

(g) Use the Chart Recorder to capture the transient response of the closed-loop system. Use a sampling rate fast enough to adequately characterize the response.

Repeat the above with one magnet under the copper disc. Estimate the time constant, and steady-state error in each case and compare the values to the estimates you made in your pre-lab assignment.

Write-Up: Record your predicted and experimental results on the summary sheet. Please include plots of your data.

Lab 3 Summary Sheet

Let τ be the closed-loop time-constant (sec), and Δ be the fractional steady-state velocity error, that is

$$\Delta = \frac{\Omega_{desired} - \Omega_{ss}}{\Omega_{desired}},$$

and fill in your results below:

	No viscous friction		With a single magnet	
	PREDICTED	MEASURED	PREDICTED	MEASURED
τ (sec.)				
Δ				