2.004 Dynamics and Control II
Spring 2008

Massachusetts Institute of Technology
Department of Mechanical Engineering

2.004 *Dynamics and Control II*
Spring Term 2008

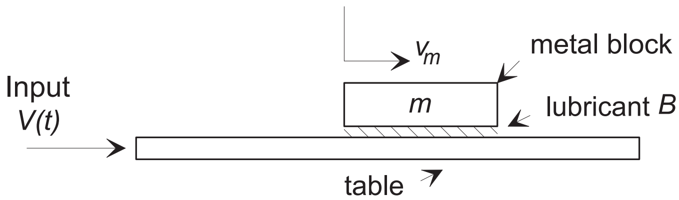
Problem Set 5

Assigned: March 7, 2008 Due: March 14, 2008

Reading:

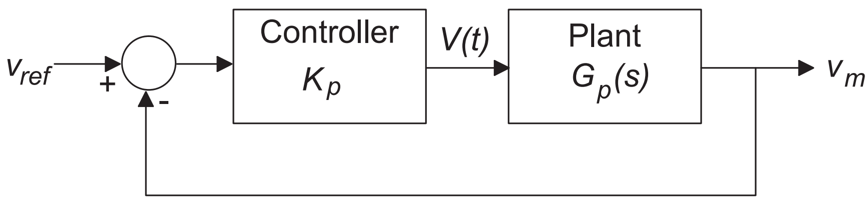
- 2.004 Class Handout: *Modeling Part 1: Energy and Power Flow in State Determined Systems*
- 2.004 Class Handout: *Modeling Part 2: Summary of One-Port Primitive Elements*
- 2.004 Class Handout: *Introduction to the MATLAB Control Systems Toolbox*

Problem 1: A metal block of mass m sits on a table. Vibrations in the floor cause the table to move horizontally with a velocity $V(t)$. A thin film of lubricant allows the block to slide on the table with an effective viscous frictional coefficient B, as shown below.

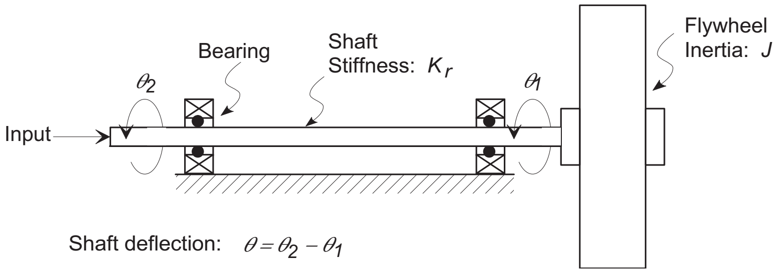


(a) Develop a transfer function $G_p(s)$ for the system, assuming that the output is the mass velocity v_m.

(c) The velocity of the mass is to be controlled with a closed-loop proportional system as shown below. Assume we have a sensor that measures the velocity, and that the controller, with gain K_p produces the velocity command directly. Determine the closed-loop transfer function.



Problem 2: (Modified from Problem Set 4.) Consider the rotational system shown below:



The torsional stiffness of a cylindrical shaft of diameter D and length l is

$$K_r = G\frac{\pi}{32}\frac{D^4}{l}$$

where G is the shear modulus of the shaft material. A steel shaft, 5 m long and 5 cm in diameter, drives a steel cylindrical flywheel with a 30 cm diameter and a thickness of 5 cm. Steel has a density of $\rho = 7.8$ gm/cm^3 and a shear modulus of $G = 83$ GPa. Assume that each of the bearings exhibits a rotational viscous frictional coefficient $B = 0.1$ N-m-s/rad. The input is a torque source $T_s(t)$.

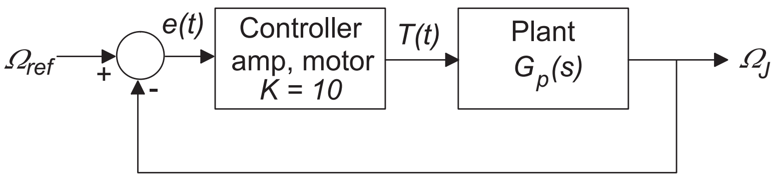
As a controls engineer you have been asked to design a closed-loop controller to maintain the flywheel at a constant speed under varying load conditions. In order to do this you need a transfer function model of the plant:

(a) What are the values of the shaft stiffness K_r and the flywheel moment of inertia J.

(b) Derive the transfer function (with numerical coefficients) relating the flywheel angular velocity Ω_J to the input T_s.

(c) Use MATLAB's *Control Systems Toolbox* (see the handout) to enter the plant (as a transfer function) and plot its step response. From your plot, estimate the "rise-time" (which is defined as the time for the response to change from 10% to 90% of the final value.

(d) Assume that we have a sensor that measures the angular velocity Ω_J with unity gain (ie it measures the angular velocity exactly). Assume also that we have a proportional controller, servo amp and motor, and that we form a closed-loop control system to regulate the flywheel speed. The controller, amp, and motor have a combined gain of 10 Nm/volt, that is the motor torque is $T(t) = 10e(t)$N/m, where $e(t)$ is the error.



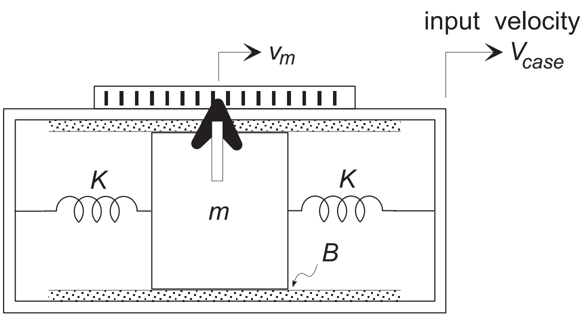
Use MATLAB's `feedback()` function (see the handout) to find the closed-loop transfer function. As in part (c), plot the step response, and estimate the closed-loop rise time.

(e) Comment on the effect of the proportional control on the system rise time.

Suggestion: You should be able to solve all of the MATLAB parts of this problem in one computer session.

<u>**Problem 3:**</u> (A deliberately vague problem statement.) A tugboat tows a heavy barge at the end of a long elastic cable in smooth water. The tug's propellers generate a controlled propulsive force, and hydrodynamic drags may be represented by linear viscous drag effects. Generate a third-order model of this system, and show the structure on an impedance graph. Define the impedance of each element on your graph. Briefly discuss why you included the elements you did. Write continuity equations for each node on your graph but **do not** solve the system.

<u>**Problem 4:**</u> Seismometers are used to measure the motion of the earth's surface. A schematic drawing of a simple seismometer is shown below. A proof mass is suspended in springs and slides horizontally on a viscous friction material. The relative displacement of the proof mass m with respect to the instrument case is used as a measure of the severity of an earthquake.

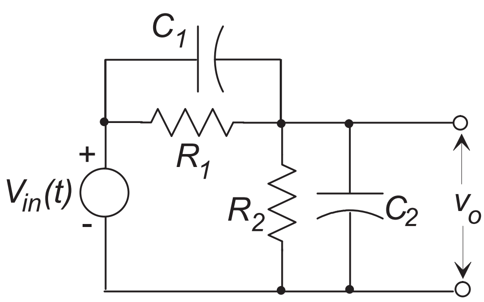


(a) Construct a linear graph model of the system.

(b) How many independent energy storage elements are there?

(c) Derive the system transfer function relating the relative displacement of the proof mass m with respect to the instrument case to the earth's earthquake velocity v_{case}. Use the mesh equation method to do this. This simple enough to do by hand, but you may use any computer tools you wish.

<u>**Problem 5:**</u> Find the transfer function $V_o(s)/V_{in}(s)$ of the RC filter shown below:



Now suppose $R_1 = 50$ kΩ, $C_1 = 1$ μfd, $R_2 = 100$ kΩ, $C_2 = 0.5$ μfd. Evaluate the transfer function with these values, and write the differential equation in this case. Can you explain what has happened to the order of the system?