

MITOCW | R4. Free Body Diagrams

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PROFESSOR: All right, you know the drill from the previous couple of weeks. Get your piece of paper out, and write down what you think were important concepts for the last week or so.

OK. Let's see what you have on your papers.

AUDIENCE: Center of mass can be used to find the total force.

PROFESSOR: So the concept is center of mass and things you can use it for. So I'm going to generalize that to "systems of particles." And what you've just said is, center of mass is one thing that comes from that.

Anybody, on this subject of systems of particles, anybody else have something?

AUDIENCE: Angular momentum?

PROFESSOR: Angular momentum. So what about it?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, so this concept of the summation of the h_i 's, equal to H at the system, and that you can use that. Anything else about systems of particles that we learned?

AUDIENCE: Newton's third law between the particles?

PROFESSOR: Yeah, Newton's third law can help to support things out, and a consequence-- you found center of mass, that's useful, but you take the derivative of the center of mass, what do you find out? rG here, dot times the total mass is what?

AUDIENCE: Momentum?

PROFESSOR: Yeah, it's momentum, but it's momentum of what?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Well, it's the momentum of the entire system, right? And so this is the summation of the Pi. That's kind of important. And finally, take another derivative, what did you learn about that? Pardon?

AUDIENCE: Forces?

PROFESSOR: What kind of forces?

AUDIENCE: External.

PROFESSOR: All the external forces are in the system. All of them. So powerful things came out of thinking about center of mass.

All right, what's something else we did in the week that wasn't around systems of particles? Yes. Actually, say your name when I call on you, I'm gonna try to learn more of your names.

AUDIENCE: Christina.

PROFESSOR: Christina, right.

AUDIENCE: So, when you're looking at equations, for example, like the force external is equal to the change in angular momentum. Looking at what pieces go where, so if you have a defined moment, does it go on this side or that side?

PROFESSOR: OK, and I'd put that in the category of free body diagrams, are you thinking of? So, constructing free body diagrams. OK, what else?

AUDIENCE: Eddie.

PROFESSOR: Eddie?

AUDIENCE: The two different methods for finding torque.

PROFESSOR: So, let's go a little further with torque. What do you use the torque for? What do you want to find torque for? To get-- let's generalize this to equations of motion, so both for forces, and moments. Two different kinds of problems that we are able to do by being able to find the torques. Anything else? It's important this week.

AUDIENCE: [INAUDIBLE]?

PROFESSOR: Yeah, I mean, this isn't a math course. Yeah, you've got to be able to solve differential equations, yeah. Maybe you're actually onto something here, but let's put it in the context of what kind of equations we work on a lot on this week. What kind of equations of motion? I don't mean like second order linear, I mean we're doing dynamics. We're doing dynamics of what kind?

AUDIENCE: Acceleration?

PROFESSOR: Yeah, but also, a lot of examples this week, were they rotational problems or translational problems?

AUDIENCE: Rotational?

PROFESSOR: Right, kind of a heavy emphasis this week on angular momentum, right? So, using-- That's a pretty good list. My own list had most of that on it. Let's see where my list was. Center of mass, \dot{r} , \ddot{r} . I wrote these as equations. So this thing in particular, the summation of the external torques with respect to some point, for a particle, is $\frac{dh}{dt}$, plus v_A cross P . And we were just beginning to do sums of collections of particles, rigid bodies, where you have maybe more than one particle, the summation of the torques is the H -- capital H -- for the summation of the h of i 's, $\frac{dh}{dt}$ plus v_A in o cross, and here's where we learned something up here that you can do down here that was really helpful. What crossed with what?

AUDIENCE: P .

PROFESSOR: Which P ?

AUDIENCE: [INAUDIBLE].

PROFESSOR: This one up here, right? And that's actually this. So this is cross P of the center of mass of the system. OK good, that's a pretty good list. So now, let's do a problem.

So here's the problem. It's a rigid rod, with a mass on the end, and the rod's massless just to make it easy for the moment. But this is now a rigid body. There's a torsional spring here. And the torsional spring resists angular displacement with a torque that's equal to the spring constant times the theta that you push it through. So it generates a moment, a restoring moment. When you push it away, it tries to push this thing back. Gravity is also acting, so it's a pendulum as a rigid body and a torsional spring.

So the first step-- I want you to eventually find the equation of motion. And I want you to start by doing a free body diagram. But before you go to work, I'm going to give you a little quick mini-lecture on free body diagrams.

So, the first thing you do is assign coordinates. Maybe draw is not-- really assign the coordinates for the problem. Determine the number of degrees of freedom. We haven't talked about that much yet. It will become much more important in the subject. The number of degrees of freedom-- but I'll say it once, and will do much more with this-- the number of degrees of freedom is the same thing as the number of independent coordinates you need to completely describe the motion. And the number of independent coordinates that you need is equal to the number of equations of motion that you'll get.

So this problem, how many coordinates does it take-- and this is planar motion, it's confined to the board. How many actual coordinates does it take to completely describe the position of this thing? I see a one, how about-- anybody else? What's the one? What would you choose?

AUDIENCE: I think I'd probably go with polar.

PROFESSOR: Yeah, well what coordinate itself? What is the single coordinate you would use to define the motion of the system? I hear a theta. Everybody agree? Theta of t, if you can figure out theta of t, you know this position of this thing for all time. That's all

you need, is one degree of freedom, one coordinate, and in terms of choosing the coordinate system to use, basically you were saying, use what?

AUDIENCE: Polar

PROFESSOR: Polar coordinates centered here. But-- there's your inertial system, but you're going to go with this point. This is our A point, and is it moving? So if we use expressions like this, what will happen to that term? It goes away.

AUDIENCE: So if you were using Cartesian, would it be two degrees of freedom?

PROFESSOR: Nope. And we'll get to that. If I divert to that, I won't finish what I've got planned for today. It's important and we're going to come back to that. Has to do with constraints.

So you assign the coordinates, determine the number of degrees of freedom, assign positive values to all rotations, displacements, velocities, linear velocities, and rotational velocities, in the problem. Positive values of those. And from that, you deduce the direction of the resulting forces and moments. And this will help you get the signs correct, especially when you have multi-body problems like two masses with a spring in between them. Which way do the forces go on each mass?

In this problem, there's only one coordinate, so what we're really saying is assign a positive theta, and assume a positive theta dot. What are the resulting forces that end up in your free body diagram? So now, I want you to draw the free body diagram for this thing, and up here, final statement, put in all forces and moments. Don't leave out anything, because you know they're not going to matter. Because it'll get you in trouble as the problems get more complicated. OK, draw your free body diagrams.

OK, so we're going to get in groups of three or four, and compare notes here. You guys are kind of a natural group-- check each other's stuff and come up with a final.

AUDIENCE: We were having a debate about whether or not you were asking us for the free body diagram of the system, or if you were asking us for the free body diagram of

the mass itself.

PROFESSOR: Yeah, so are there moments and things involved up here? Like that torsional spring?

AUDIENCE: When you are including the entire system there, yeah.

PROFESSOR: OK, so this is a rigid body. I kept saying this is a rigid body. Happens to have a single mass point in it, but it's a whole, single, rigid body. And do the free body diagram for the rigid body.

PROFESSOR: Can you show me something? Where are you at-- what's your best shot at this so far?

AUDIENCE: The free body diagram needs [INAUDIBLE]. And we say the spring force is producing the moment.

PROFESSOR: So what's your body that you're working with? What's the definition of a rigid body?

AUDIENCE: Mass on the end of the rod?

PROFESSOR: OK, so you're still thinking about the particle. But if you don't think about the particle, it's kind of hard to get that torsional spring involved. So you need to think of this as a single, whole body. One whole, rigid body. And now, just draw a stick in here, and put the forces on it. Doesn't matter where the mass is for the purpose of the free body diagram. That comes into the--

AUDIENCE: Couldn't you just use the r cross force? And get the force in the ball from the--

PROFESSOR: So you're getting into the equation of motion part of it. This is only about assuming this thing has some dynamic moment in time, and draw the external forces and moments that act on the object. And the object is this full-length stick.

AUDIENCE: So in that case, we just have the spring force acting, in opposition to [INAUDIBLE] the weight?

PROFESSOR: You have weight, for sure. You have the spring, puts a moment on the system, and

there are yet potential other forces.

PROFESSOR: I'll give you a couple minutes, because there's a little confusion around whether it's a particle or a rigid body. It's a rigid body. You've got to deal with the whole body. All right, let's see what you've got. Where's your free body diagram?

AUDIENCE: This right here.

PROFESSOR: OK, tell me what you have for forces and moments, just point to each one of 'em and say what it is.

AUDIENCE: So, we have the weight of the this end, and we assume this is massless.

PROFESSOR: Yup, but it's rigid, so it's all one system now, you have one rigid body.

AUDIENCE: So the center of mass is right here, because there's-- [INAUDIBLE], and then we have reaction forces at the pin, and then we have the moment from the spring there.

PROFESSOR: OK.

AUDIENCE: We have the balls moving forever--

PROFESSOR: So, you've gotta convince one another who's right, here, so--

AUDIENCE: Well, it depends on which way the ball's moving.

PROFESSOR: So, that's why there is a little system. There's a rubric that I-- I recommend you just use the system. This is basically a system for getting equations of motion, and this is important, that one. So then if you know it's positive, then you can figure out which direction the reaction is.

All right, I hate stopping all the fun. You guys are doing well. There is great value in talking to one another, and convincing one another of what the right way to do something is. You'll have two different points of view, and talking it out, It's amazing how fast you can make progress. So do that, do that when you're home solving problems.

So what was the assignment here? Draw a free body diagram. Let's do it. So it's a rigid body. All one body, even though the mass is all concentrated down here, the forces-- what are forces that are on this thing? Give me one. Wait, mg , right? OK, what else?

AUDIENCE: Normal force?

PROFESSOR: Normal force. Where?

AUDIENCE: On the top.

PROFESSOR: OK, so you're talking about a reaction force up here? And have you chosen a coordinate system? We already kind of decided, didn't we, on polar coordinates for this, right? So are the reaction forces known? To start with, no. So what's the cleverest way to assign them on your free body diagram?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Well, and associated with your coordinates, right? So, I would have two reaction forces up here, one in each of which directions?

AUDIENCE: Radial.

PROFESSOR: Radial, and?

AUDIENCE: Tangential.

PROFESSOR: OK, and the positive radial, positive tangential, like that, right? So I would say, I have an unknown F_θ force, and I have an unknown F_R force, and I draw them in the positive directions because I have no clue. So just make them positive. You assume direction for them. Force is here, g there, what about the tension in the string?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Does it belong in this problem? Why not? The rigid body-- it's an internal force. It's

the little $\hat{f}_i \hat{f}_j$ thing, it's those things that cancel internally, right? So you don't have to deal with it. It's not an external force to the rigid body. So this is great. The next step in this is to find an equation of motion. Gotta to be speedy about this. So, what--

AUDIENCE: [INAUDIBLE].

PROFESSOR: Pardon?

AUDIENCE: Moment.

PROFESSOR: Moment. You'd use moment to do it. About what point?

AUDIENCE: About the point up at the top.

PROFESSOR: All right, so if we call that A , you'd use angular momentum, and torques, and that sort of thing? Is that what you're saying? Moments about it?

AUDIENCE: Oh, I was just wondering if you were gonna draw it up on there.

PROFESSOR: No, I'm gonna have you do it yourself. Do it. So now--

AUDIENCE: Well, is she talking about how there's the moment-- the resistant moment.
[INAUDIBLE].

PROFESSOR: Oh. Did we forget that one? So which direction is it? Clockwise, or counterclockwise?

AUDIENCE: It's clockwise.

PROFESSOR: All right, so there's a moment about this point that is-- we know what it is. It's in that direction that has value kt times θ . Any time you know what it is, don't just make it an unknown, write it out. You know it's that, and you know it's in this direction. And we're going to write-- how many equations of motion will we get?

AUDIENCE: [INAUDIBLE].

PROFESSOR: And we've chosen polar coordinates. You could do this using forces. Just Newton's second law. But then you have to solve for what? If you write force balance, F

equals ma , you have these up here, and you don't know them. So the reason to use torques in this problem is because, are these going to appear in your answer? No. So that's the reason to use torques. Yeah.

AUDIENCE: Is that k -- [INAUDIBLE].

PROFESSOR: This is k sub t to distinguish it as a torsional spring.

AUDIENCE: So [INAUDIBLE] multiplied by L --

PROFESSOR: No, it doesn't have an L , it's right here. It's a spring who gives a moment resistance to being deflected. kt theta is units of torques. And this isn't units of force per unit displacement, this is units of torque per radian. OK? Yeah.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Well, you have assigned polar coordinates, and my assumption was that-- I drew it right here. There is positive θ hat. Right?

AUDIENCE: So isn't our torque negative?

PROFESSOR: Yeah. But see, I've drawn-- this negative is accounted for by the direction of the arrow. So let's now write the equation of motion, and to do the equation of motion, you're going to say that the sum of the what?

AUDIENCE: Torques.

PROFESSOR: External torques. About what point?

AUDIENCE: A.

PROFESSOR: A is equal to?

AUDIENCE: $dH dt$?

PROFESSOR: And I'll use the capital H , because we're now working our way onto rigid bodies. We could have more than one point, we could have more than one mass point, $dH dt$ plus?

AUDIENCE: v.

PROFESSOR: v_A cross P , and what's that in this problem?

AUDIENCE: 0.

PROFESSOR: That guy conveniently is 0. So, do it. You can now do this problem. Figure out, write down an equation of motion, using some of external torques and-- so you're gonna have to figure out angular momentum and some torques.

I hate pulling you guys away. Lots of good thinking going on. All right, drag yourselves away, help me out here. Which groups figured out the external torques? What do you got? Tell me what to do here.

AUDIENCE: We have the one for gravity.

PROFESSOR: Tell me what it is. I wanna write the terms down.

AUDIENCE: It is negative $Lmg \sin \theta$.

PROFESSOR: Minus $mgL \sin \theta$, in what direction?

AUDIENCE: $k \hat{}$?

PROFESSOR: Others agree with that term? Hmm?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Wait a second, just one term at a time here. The gravity term, do we have the sign correct? And then we have the moment arm correct? So this is the moment arm, r cross force. Looks like minus k to me. $mgL \sin \theta$. Looks pretty good. What other torques?

AUDIENCE: The moment from the spring.

PROFESSOR: Moment from the spring. Tell me how to write that.

AUDIENCE: That's in the negative \hat{k} section, so minus $m\hat{h}$. Well, then [INAUDIBLE] kt θ .

PROFESSOR: $kt\theta$, also in \hat{k} direction. And our free body diagram, the forces produce moments. Is everybody happy with that? Are we done? Any questions about it? Why it's used the way it is? OK.

Now we need this piece of it. So what's the H ? For rigid body, the summation of all the bits-- how many bits are there? Just the one. And that's $\sum \mathbf{r} \times \mathbf{P}$. Can somebody tell me what they got for the \mathbf{p} , is the momentum of our little mass here, with respect to an inertial frame, which is also A because it's not moving. And \mathbf{r} is L . So this is $L\hat{r} \times m$, what? $L\dot{\theta}$ is its velocity, mass times velocity, and its direction? $\hat{r} \times \hat{\theta}$ is \hat{k} , so this looks like $mL^2\dot{\theta}\hat{k}$. Are we all in agreement?

dH/dt ? This \hat{k} changed direction, so the only time-dependent thing--

AUDIENCE: I think you're missing a dot.

PROFESSOR: Oh, thank you, we're going to need that. All right, the derivative is?

AUDIENCE: [INAUDIBLE].

PROFESSOR: OK, that's equal to that. It's all \hat{k} 's, you need just one equation out of this, you don't have to break it down into subcomponents. And you can collect it all on one side. $mL^2\ddot{\theta} + kt\theta + mgL\sin\theta = 0$. And that's a second order nonlinear ordinary differential equation, which you could linearize for small motion. It's just a pendulum, OK? Good.

We got one other one I want to do before the hour's out. I got a big piece of cement pipe on the back of a truck. It's not tied down. The truck's moving at 3 meters per second. And the rotation rate of this piece of pipe is 6 radians per second. And the truck's accelerating, minus $1/2$ a meter per second squared. It's actually breaking. The guy's in trouble, right? So the first thing here is-- $\omega = 6$ radians per second, does it matter, with respect to what, the truck or the ground? I haven't told you. Talk

in your groups, and give me a quick answer to that. What do you think?

AUDIENCE: Doesn't matter.

PROFESSOR: Say again?

AUDIENCE: Doesn't matter?

PROFESSOR: So Kristen thinks it doesn't matter. Anybody wanna counter that? Why not? So, you're saying that this could be omega with respect to the truck? Or it could be omega with respect to this fixed frame, and the answer will be the same in either case? Christine, right?

AUDIENCE: Christina.

PROFESSOR: Christina, alright.

AUDIENCE: So earlier, we had that problem with-- there was an r , [INAUDIBLE] and then there was another [INAUDIBLE] over here. You had to know what this omega was with respect to what, because this arm's rotating also. And so, with this, you have your one xy-coordinate system, and while the truck is translating, the truck's not gonna be in rotation itself, so it's staying in the same coordinate plane, and [INAUDIBLE].

PROFESSOR: So your argument is if you were were standing on the ground, seeing it rotating, you'd say-- you would see it rotated at the same rate as if you were riding on the truck. Everybody agree with that? Does it matter that the truck's accelerating?

AUDIENCE: [INAUDIBLE].

PROFESSOR: No, it's only whether or not the truck's rotating. If the truck's not rotating, then rotation rate seen from this frame or just a translating frame will always seem the same. Even if you're accelerating. OK, so we've done that one.

Now, I want you to find the velocity of point G, given this information. So, whip it down quickly, how you would approach this problem. See if you can solve it. G is the center of mass of that big pipe.

OK, we're gonna run out of time. Who's got an answer for me? I think they've got it sorted out. Kristen?

AUDIENCE: So, we have velocity of the truck, minus [INAUDIBLE].

PROFESSOR: All right, so let's be systematic. You have to pick some points, and did you use a rotating frame? Use an equation that requires a rotating reference frame?

AUDIENCE: No, because the translational [INAUDIBLE].

PROFESSOR: So I assume you're talking about-- we're looking at the velocity of G in o, is equal to, in general, the velocity of some point A that you're gonna have to pick, plus the velocity of G with respect to A. And this thing has two possible components, right? One which is the derivative of GA, dt, ignoring the rotation, plus-- OK. What's this term in this problem? Well, we haven't picked the point A. Where are you going to pick point A? There's dumb points, and there's good points. What?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Front of the truck. Well, it's possible. Christina, what do you think?

AUDIENCE: I couldn't-- on the bottom of this part [INAUDIBLE].

PROFESSOR: The point of contact? Is that what you're saying? Anybody else? What do you vote for? Where do you want to put A?

AUDIENCE: Point of contact.

PROFESSOR: Point of contact, OK. If you're using this equation, and you're using these pieces, you are using the concept of a rotating frame. If there's any omega in the problem, you'd better have a rotating frame somewhere. Unless you're using polar coordinates, which is sort of a degenerate rotating frame. So at this point, we're going to make A here. And you have a coordinate system, little x_1 y_1 , and attach the A. So you have an A x_1 y_1 , here z_1 .

And is it rotating? Is it a rotating coordinate system? Well, if it's not, then you can't

do this. This assumes-- this term here is the velocity at which this point and this point are moving relative to one another, as if you were sitting on the pipe. Ignoring the-- you don't see the rotation, you're moving with the pipe. It's assuming you have then put a reference frame on the pipe that moves with the pipe. So does the little reference frame, reference frame $Axyz$ move with the pipe? It better, or you can't use the equation.

So this is attached to the pipe. And as it rolls, you have to allow that xyz frame to move with it. OK. So in that frame, what's this term? This is 0 because the two points are fixed in the pipe. And what's this velocity?

AUDIENCE: 3.

PROFESSOR: 3 meters per second in what direction? We'll do it in an inertial frame if we want. And then what's this term? So it's 6, and we said it doesn't matter what frame it is, it's all the same rotation rate, radians per second. In what direction? \hat{k} cross, the radius is capital R , what's this direction? And now this is at an instant in time. At this particular instant in time, you've cleverly drawn the xyz system so it lines up with them after so it makes the problem easy.

So at the moment, it's either in the big \hat{J} or in the little \hat{j} . It's easiest if you do it in big-- call it big J , because then you're done. So, \hat{k} cross J , I think negative I . And so this should work out to be 3 meters per second positive I . $1 \text{ and } 1/2$ times 6 is 9 and a minus. 9 meters per second in the I direction. So you are at minus 6 meters per second.

AUDIENCE: Isn't the radius from point G ? [INAUDIBLE].

PROFESSOR: Wouldn't it be what?

AUDIENCE: Does it matter if it's negative?

PROFESSOR: Ah, it matters a lot. You have to exercise great care when you pick the R vector. And the R vector goes from your origin to the point you're talking about. So the origin, that's your frame, and that's the point. And when you draw the arrow, the R

vector, it goes from A to G. That's its direction. And it has-- its length is capital R long. Yeah, that's a really, really important point, which way the arrows go. OK, and so now we're done with that.

Let's see, we've got a minute or two. Let's talk about the acceleration. Let's do the acceleration of this problem, acceleration of G in o, and this is just an exercise in remembering the terms. Full 3D vector equation for accelerations. What's the first term here?

AUDIENCE: [INAUDIBLE].

PROFESSOR: How about before we even get into-- we're not in polar coordinates in this.

AUDIENCE: The acceleration of A in o?

PROFESSOR: All right, let's just work it out. I start with this one myself, acceleration of A with respect to o. What's another term?

AUDIENCE: Acceleration of G with respect to A.

PROFESSOR: So the acceleration of G with respect to A, but no rotation, right? That term. Plus, how many more terms do we have to go? Three, OK. Give me one. Somebody else?

AUDIENCE: [INAUDIBLE].

PROFESSOR: What's your name?

AUDIENCE: Stephen.

PROFESSOR: Stephen. You've got a 2ω , and ω is-- let's get this unit vector in here. Well, actually let's not. $2\omega \times r$, let's just do that. OK? Give me another term. Somebody else?

AUDIENCE: $\omega \times$ [INAUDIBLE].

PROFESSOR: $\omega \times \omega \times r$. And got another one.

AUDIENCE: [INAUDIBLE].

PROFESSOR: Yeah, and in fact, we'd better be a little more careful than that. So it's really the velocity of G with respect to A. And does it have rotation in it or not? This is the no rotation. This is the speed at which the points are moving apart from one another, right? $\Omega \times \Omega \times r$, we've got-- one, two-- I think we need another term. $\Omega \cdot \times r$. And the r is r_{GA} , this r is GA. OK, so what's the acceleration of this?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Nope.

AUDIENCE: Negative half.

PROFESSOR: So this one's minus 1/2. What's this one? So you're in the frame of the pipe now, and this is the speed at which the acceleration of G with respect to A. It doesn't move, it's fixed length, right? This one, what's the velocity of G with respect to A, without rotation involved?

OK, and this one is $\Omega \cdot k \times$, and this is our, again, $\Omega \cdot k \times$. And this RG, it's R in what direction at this instant? J, right? So we could work that out. And what about this one? What direction is $\Omega \cdot$ in? $k \times R$, at this instant, J again. Do we know that value? No. That might be something we have to figure out. So you could now crank this out. That'd be the acceleration of that point, but you'd find out that we don't have that. Now, how would you go about finding out what $\Omega \cdot$ is?

AUDIENCE: Equations of motion?

PROFESSOR: You get the gold star. Equations of motion. You need to come up with equations of motion. And how many equations of motion would we end up writing? This is equivalent to asking how many degrees of freedom are there. I hear 2, I hear a 1.

Now, there's a lot of nuances to this discussion. Anytime you're given a fixed value

for something, that coordinate is constrained, that parameter's constrained. We're given the motion of the truck. It's completely specified. You could just substitute numbers in for those things. I mean, like this, it pops up as $1/2$.

I think this thing will boil down to one equation. And you need to pick some set of coordinates to describe it. So if I'm looking for ω , I'd probably work with torque about that point. Isolated to just this object, and this is some of the torque around this point, and see what happens. Some of the external torques gotta be equal to what? $DH dt$, and then you have to sort out this term. So in this problem, you know-- what's the direction of this velocity? \hat{i} . What is the unit vector associated with the linear momentum of that pipe? What direction is the linear momentum of the pipe? What's the velocity? What is this answer? What's the momentum of that pipe? Mass times that velocity, right? Remember, the velocity of the system, the pipe may have all its mass around the rim, but the velocity of the momentum of the pipe is the total mass of the pipe times the velocity of its center of mass. And that's the velocity of the center of mass. So its linear momentum has what unit vector assigned with it? \hat{i} . And what is the unit vector was associated with v_A ? \hat{i} . \hat{i} cross \hat{i} is?

AUDIENCE: 0.

PROFESSOR: So this is one of those cases where they're parallel paths, and that term drops out. So you could come up with it very quickly and easily. Equation of motion. OK. Very good. See you on Tuesday. Next Tuesday lecture time will be review. I think it's gonna be more like one of these sessions, and then quiz Tuesday night.