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**PROFESSOR:** Because some of this course was shot in two different years, two different notation systems were used. And I'm going to explain both of them so that when you encounter them in the videos of the recitations of the videos of the lectures, you'll be able to use either of the notation systems interchangeably. So the two notation systems would refer to how we explain position vectors, velocity vectors, and vectors of any kind that might be associated with translating and rotating reference frames.

So in this diagram, I've got a rigid body. And attached to that rigid body is a reference frame. I'll call it XYZ. Attached and moves with the body. And that's reference frame AXYZ.

And the whole system is translating and rotating in an inertial frame O, capital X, capital Y, capital Z. And I need to be able to describe the position and the velocity of this rigid body, and a point on this rigid body, which I'll call B, which might actually even be moving with respect to the rigid body.

So the position of this reference frame in System I-- this is Notation System I-- we designate as  $R^O$  of A in reference frame O. And the O is in superscript that precedes the R.

Point B is  $R^O$  of B in O. And this vector is  $R^O$  of B with respect to A. And we write it  $B^O$ . And with respect to A is a superscript. So this essentially, the superscript version of the notation.

We do the same thing in a slightly different way in which we say with respect to is a slash. So A, with respect to frame O, is written  $R_{/O}$ . Point B is  $R_{/O}$ . And the vector that goes from A to B is  $R_{/O}$  of B/A. So the two are exactly equivalent.

And we'll go one step farther. And that is to take the time derivative of this vector  $\mathbf{B}$  and use it to derive expressions for velocities in a rotating and translating frame. The position  $\mathbf{R}_B$ , for example, using this notation, with respect to  $O$ , is  $\mathbf{R}_A$  with respect to  $O$  plus  $\mathbf{R}_B$  with respect to  $A$ .

So it's just a vector sum, this vector plus this vector equals that vector. And we want to take the time derivative of this expression for  $\mathbf{R}_B$  with respect to  $O$ . And it will give us an expression for the velocity of point  $B$  with respect to the  $O$  frame.

And here, I've written this out in both notation systems. So in the notation system where we use slash  $O$  as the with respect to, the velocity of  $B$ , with respect to  $O$ , would be the velocity of that frame, translational velocity of  $A$  with respect to  $O$ , plus the time derivative of the vector  $\mathbf{R}_B$  with respect to  $A$ . And that time derivative must be taken in the inertial frame, So  $/O$ .

This term expands into two pieces. So this is equal to, again,  $V$  of  $A$  with respect to  $O$ , but now a derivative of  $\mathbf{R}_B$ , with respect to the  $A$  frame. This is as if you were sitting on that rigid body. Is that vector getting any longer or shorter? Plus the rotation,  $\boldsymbol{\omega}$  of the body with respect to  $O$  cross-product with  $\mathbf{R}_B$ .

I've left out underscores here to emphasize that these are vectors. But these are all vectors. And in the alternative notation system, the  $/O$ s become superscripts. So the velocity of  $B$  and  $O$  is the velocity of  $A$  with respect to  $O$  plus the velocity as seen from the point of view of being on the rigid body plus-- this is the contribution to velocity as seen in the inertial frame caused by the rotation of the body.

And that's the two different notation systems. And you'll see these used on solutions to problems. You'll see them in either of the two frames are either of the two notation systems. And you will see these notation systems used in lecture and in these recitations.