2.003SC Engineering Dynamics
Quiz 3 Solutions

Problem 1 Solution:

Modal Analysis Solution:

a) The natural frequencies of the system may be computed directly from the elements of the modal mass and stiffness matrices.

$$\omega_1^2 = \frac{K_1}{M_1} \Rightarrow \omega_1 = \sqrt{\frac{190.4875}{48.981}} = 1.972 \text{ r/s}$$

$$\omega_2 = \sqrt{\frac{210.5125}{11.5579}} = 4.269 \text{ r/s}$$

b) Again, directly from modal values:

$$\zeta_1 = \frac{C_1}{2\omega_1 M_1} = \frac{9.5244}{2(1.972)(48.981)} = 0.0493$$

c) $\begin{Bmatrix} q_{10} \\ q_{20} \end{Bmatrix} = 0, \ \begin{Bmatrix} \dot{q}_{10} \\ \dot{q}_{20} \end{Bmatrix} = \underline{u}^{-1}\dot{\underset{\sim}{x}}(0) = \begin{bmatrix} 0.525 & -0.0499 \\ 0.475 & 0.0499 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.525 \\ 0.475 \end{Bmatrix}$

d)

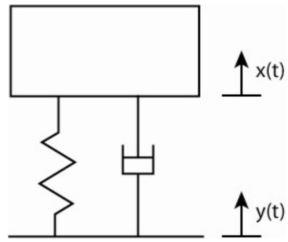
$$\underset{\sim}{x} = \underline{u}q = \begin{Bmatrix} 1.0 \\ -9.5125 \end{Bmatrix}^{(1)} \frac{\dot{q}_{10}}{\omega_{1d}} e^{-\zeta_1 \omega_{1d} t} \sin(\omega_{1d} t)$$

where $\dot{q}_{10} = 0.525, \ \omega_1 = 1.972 \text{ r/s} \approx \omega_{1d}$

$$\frac{\dot{q}_{10}}{\omega_{1d}} = 0.266$$

Problem 2 Solution:

Vibration isolation:



$$f_M = 3 \ \text{Hz}$$
$$\zeta = 0.13$$

a) The total equivalent spring constant for springs in parallel is the sum of the individual ks:
$K_{eq} = 4k$

b) For a linear system, steady state response:
frequency in=frequency out
$f_{in} = f_{out} = 10Hz$

c)

$$\left|\frac{x}{y}\right| = \frac{(1 + (2\zeta\frac{\omega}{\omega_n})^2)^{1/2}}{\left[(1 - \frac{\omega^2}{\omega_n^2})^2 + (2\zeta\frac{\omega}{\omega_n})^2\right]^{1/2}}$$
$$= \frac{(1 + .867^2)^{1/2}}{[(1 - (\frac{10}{3})^2)^2 + (0.867)^2]^{1/2}} \qquad \frac{f}{f_n} = \frac{\omega}{\omega_n} = \frac{10}{3}$$
$$= \frac{1.32}{10.148} = 0.130$$

$$\boxed{\frac{x}{y} = 0.130}$$

Problem 3 Solution:

$r = 45$ RPM
$I_{zz,G} = 100\text{kg-}m^2$
$\tau(t) = \tau_0 cos\,\omega t,\ \omega = 6\pi$ r/s
$k_t = 1600\pi^2$N-m/r
$c_t = 8\pi$ (N-m-s)/r

a).

$$\sum \tau_{/o} = \tau_o \cos\omega t\hat{k} - c_T\dot{\theta}\hat{k} - k_t\theta\hat{k} = I_{zz,G}\ddot{\theta}\hat{k}$$
$$\Rightarrow I_{zz,G}\ddot{\theta} + c_T\dot{\theta} + k_t\theta = \tau_o\cos(\omega t)$$

b).

$$\omega_n = \sqrt{\frac{k_t}{I_{zz,G}}} = \sqrt{\frac{1600\pi^2}{100}\frac{\text{N-m}}{\text{kg-}m^2}} = 4\pi \text{ r/s}$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$
$$= .99995\omega_n$$
$$\approxeq \omega_n$$

$$\boxed{\zeta = \frac{c_t}{2I_{zz}\omega_n} = \frac{8\pi}{2\times 4\pi\times 100} = 0.01}$$

c). $\omega_{osc} = \omega = 6\pi$ r/s

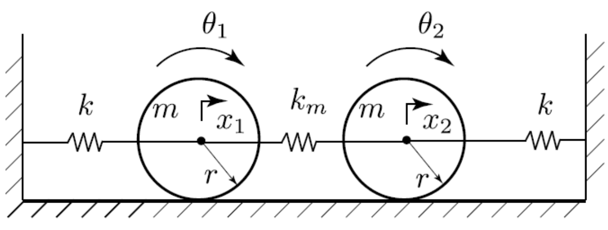
d). $\frac{\tau_0}{k_T} = 0.2$ radians

$$|\theta| = |\tau_0|\cdot|H_{\theta/\tau}| = \tau_0\frac{1/K_T}{\left[(1-\frac{\omega^2}{\omega_n^2})^2 + (2\zeta\frac{\omega}{\omega_n})^2\right]^{1/2}}$$

$$\theta = \frac{\tau_0}{K_T}\left[(1-\frac{\omega^2}{\omega_n^2})^2 + (2\zeta\frac{\omega}{\omega_n})^2\right]^{-1/2}$$, which when evaluated at $\omega/\omega_n = \frac{6\pi}{4\pi} = 1.5$ yields:

$$= \frac{0.2\text{ rad}}{[(1-1.5^2)^2 + (2(.01)(1.5))^2]^{1/2}}$$
$$= \frac{.2}{[1.56 + 0.0009]^{1/2}}$$
$$= \boxed{0.16 = \theta}$$

Problem 4 Solution:



a) The system has two degrees of freedom for the no-slip condition.
If has four degrees of freedom if slip is allowed.

b) Two generalized conditions are required.
I choose coordinates x_1 and x_2, where
$x_1 = r\theta_1$, $x_2 = r\theta_2$.

c)

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}I_{c_1}\dot{\theta}_1^2 + \frac{1}{2}I_{c_2}\dot{\theta}_2^2$$

for uniform disk, $I_c = mr^2/2$.

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}\frac{m\,r^2}{2}\frac{\dot{x}_1^2}{r^2} + \frac{1}{2}\frac{m\,r^2}{2}\frac{\dot{x}_2^2}{r^2}$$

$$= \frac{3}{4}m\dot{x}_1^2 + \frac{3}{4}m\dot{x}_2^2$$

$$V = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2 + \frac{1}{2}k_m(x_2 - x_1)^2$$

d) Find EOMs:
From Lagrange $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = \frac{3}{2}m\ddot{x}_1\ ,\ \frac{\partial T}{\partial x_1} = 0$$

$$\frac{\partial V}{\partial x_1} = kx_1 - k_m(x_2 - x_1)$$

$$\Rightarrow \boxed{\frac{3}{2}m\ddot{x}_1 + (k + k_m)x_1 - k_m x_2 = 0}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = \frac{3}{2}m\ddot{x}_2\ ,\ \frac{\partial T}{\partial x_2} = 0$$

$$\frac{\partial V}{\partial x_2} = kx_2 + k_m(x_2 - x_1)$$

$$\Rightarrow \boxed{\frac{3}{2}m\ddot{x}_2 - k_m x_1 + (k + k_m)x_2 = 0}$$

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