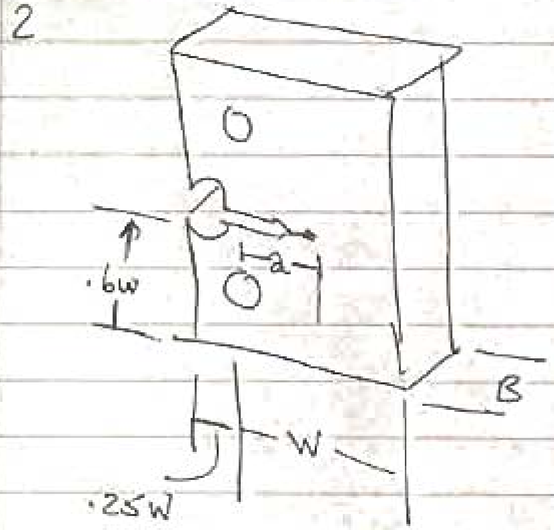
2.002
QUIZ II SOLUTIONS
(ROUGH)

2.1

2



Given: $\sigma_y = 1400$ MPa
$K_{IC} = 100$ MPa$\sqrt{m}$
(est.)

estimated plastic
zone size:

$$r_{IC} = \frac{1}{2\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2 = \frac{1}{2\pi}\left(\frac{100}{1400}\right)^2 m$$

$$= 0.81 \text{ mm}$$ 5

SSY: in-plane dimension $\geq 15\, r_{IC} = \underline{12.15 \text{ mm}}$
a, w-a, etc

Plane Strain: $B \geq 15 r_{IC} = 12.15$ mm
5

2

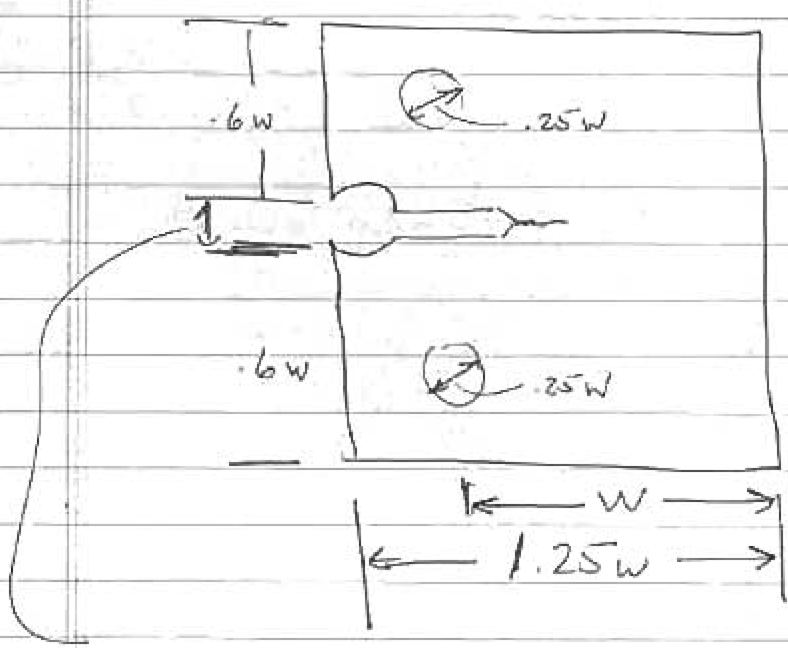
Evidently, choose plate thickness $B \sim 12.5$ mm $\simeq 1/2$ inch as minimum:

SIMILAR CONS

$a/W = \frac{1}{2} \Leftrightarrow a = W - a$ so

$$\frac{W}{2} = a > \sim 12.5 \text{ mm}$$

$$W \gtrsim \sim 25.4 \text{ mm} = 1 \text{ in}$$



ASSUME SLOT WIDTH $\simeq .1w$ to $.2w$

$$\text{Gross Area} = (1.25W)(1.2 + .1 \text{ to } .2)W$$

$$\simeq 1.25^2 W^2$$

$$\text{Gross Volume} \doteq B W^2 (1.25)^2 = (12.5 \text{ mm})(1.56)(25 \text{ mm})^2$$

$$\rho = 7.8 \times 10^3 \ kg/m^3$$

$$= 7.8 \ g/(cm)^3$$

$$V = 12.6 \times 10^3 \ (mm)^3$$

$$= 12.6 \ (cm)^3$$

ξ

$$M = \rho V = 12.6 \ cm^3 \times 7.8 \ g/cm^3$$

$$= 98 \ g = .098 \ kg \approx .1 \ kg \ x$$

$$2.2 \ lbf = (1 \ kg) \cdot g \Rightarrow .22 \ lbf \approx \frac{16}{5} \doteq 3 + o$$

3.1 plastic strain increment:

$$d\varepsilon_{ij}^{(p)} = \frac{3}{2}\, d\bar{\varepsilon}_p \, \frac{\sigma'_{ij}}{\bar{\sigma}}$$

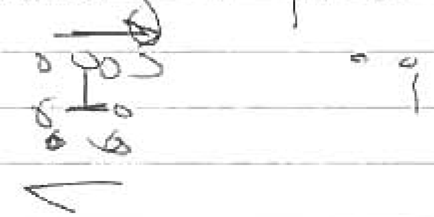
(1) Zero Plastic Volume change

$$0 = \frac{\Delta V^{(p)}}{V} = \sum_{i=1}^{3} d\varepsilon_{ii}^{(p)} = \frac{3}{2}\frac{d\bar{\varepsilon}^p}{\bar{\sigma}} \sum_{i=1}^{3} \sigma'_{ii} \nearrow 0$$

Because the trace of any deviatoric tensor $\equiv 0$ $\left(\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\left(\sum_{k=1}^{3}\sigma_{kk}\right) \Rightarrow \right.$

$$\sum_{i=1}^{3} \sigma'_{ii} = \left(\sum_{i=1}^{3}\sigma_{ii}\right)\left[1 - \frac{3}{3}\right] = 0$$

Dislocation picture: plasticity by shearing of planes of atoms $\Rightarrow$ NO VOLUME CHANGE



(2) Plasticity "turned on" only when $\bar{\sigma} = S$

Plastic flow macroscopically independent of mean normal stress, $\sigma_m = \frac{1}{3}\sum_{i=1}^{3}\sigma_{ii}$

Since a change in σ_m alone produces NO change in Mises equivalent stress, $\bar{\sigma}$

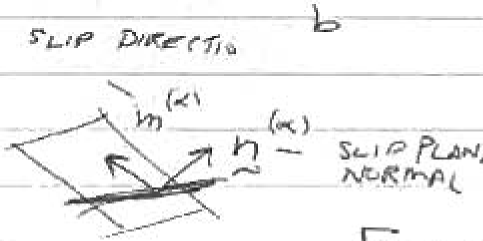
At the microscopic level, a dislocation moves only when the resolved shear force/length acting on the dislocation exceed the critical force/length, f_c

DISL MOTION : $f = f_c$

NO DISL MOTION : $f < f_c$

But $f = \tau b$, where τ is resolved <u>shear</u> stress, & b is Burgers vector.

Thus $\tau = \frac{f_c}{b} = \tau_c$ is required to move a dislocation & make plasticity



For a slip system "α",

$$\tau^{(\alpha)} = \underset{\sim}{m}^{\alpha} \cdot \left(\underset{\sim}{\sigma}\, \underset{\sim}{n}^{(\alpha)}\right)$$

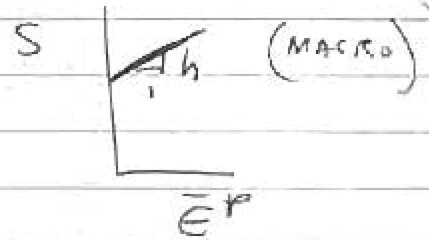
$$= \sum_{i=1}^{3}\sum_{j=1}^{3} m_i^{(\alpha)}\, \sigma_{ij}\, n_j^{(\alpha)}$$

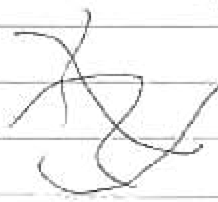
SINCE $\underset{\sim}{m}^{(\alpha)} \perp \underset{\sim}{n}^{\alpha}$,

addition of $\sigma_m \delta_{ij}$ as a mean stress change will give NO change in $\tau^{(\alpha)}$.

AND thus ~~will give~~ the motion [or not] of any dislocation will [also] be independent of the mean normal stress, as in macro equations.

(3) Strain hardening (MICRO)





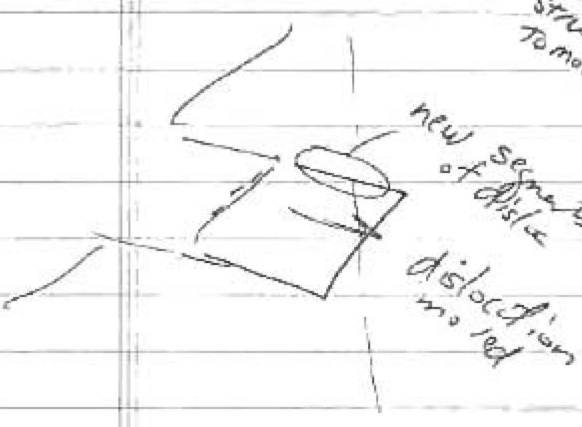
DISLOCATION Density ρ INCREASED with Plastic strain

Dislocations Act as OBSTACLES to the motion of other disl => MORE OBST => Need more stress to move

$$\tau_c = \tau_i + \tau_{ss} + \tau_{gb} + \tau_{obst} + \tau_{dis}$$

intrinsic, solid soln, grain bndr, ppt/obstach – all = independent of plastic strain

τ_{dis} ↑ & $G b \sqrt{\rho_{dis}}$ ↑ w/ plastic strain multiplic. dislocat.



(4) ON/OFF switch

MACRO:

$$d\bar{\epsilon}^{p} = 0 \text{ if } \bar{\sigma} < s$$

$$\text{or } \bar{\sigma} = s \ \& \ d\bar{\sigma} < 0$$

And incremental strain is purely elastic:

$$d\epsilon_{ij} = d\epsilon_{ij}^{(e)}$$

MICRO:

If $f < f_c$, No dislocation motion,
So NO PLASTIC
DEFORMATION
AND Therefore any
observed deformation
(for non-moving dislocations)
must be elastic only

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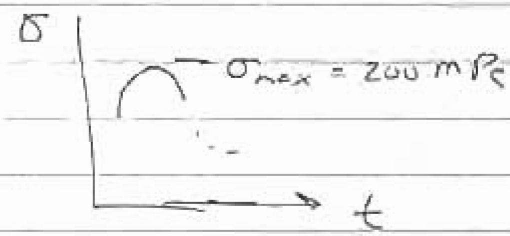


At fracture

$$K_I = \sigma_{max} \cdot Q \sqrt{\pi a_f}$$

(A)

$$= (200\,MPa)(1.12)\sqrt{\pi \times .165\,m}$$

(20 POINTS)

$$\boxed{K_I \text{ at fract} = 161.3\ MPa\sqrt{m} = K_{IC}}$$

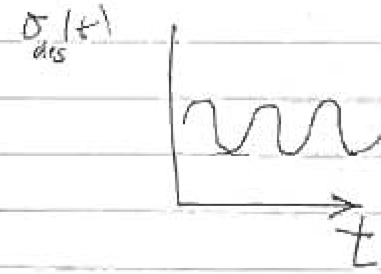
Specification

$$K_{IC} \geq 150\ MPa\sqrt{m}$$

Thus K_I at fract $> 150\ MPa\sqrt{m}$
so there is no reason to believe that the material failed to satisfy the 150 $MPa\sqrt{m}$ toughness specification

4.2

(B)



$\sigma_{max}^{(des)} = 200$ MPa
$\sigma_{min}^{(des)} = 100$ MPa

$\Rightarrow \boxed{\Delta\sigma^{des} = 100 \text{ MPa}}$

What is expected number of cycles to grow crack from $a_i = 1$ mm to $a_f = 165$ mm under the design-value of $\Delta\sigma$, namely, $\Delta\sigma^{des} = 100$ MPa

$m = 4$; $\Delta K_{I0} = 10 \text{ MPa}\sqrt{m}$; $\Delta a_0 = 3\times10^{-6}$ mm/cycle
$Q = 1.12$;

$$N_{a_i \to a_f} = \frac{a_i}{\Delta a_0}\left(\frac{\Delta K_{I0}}{Q\Delta\sigma\sqrt{\pi a_i}}\right)^m \left(\frac{2}{m-2}\right)\left[1-\left(\frac{a_i}{a_f}\right)^{\frac{m-2}{2}}\right]$$

$$= \frac{1 \text{ mm}}{3\times10^{-6} \text{ mm/cycle}}\left(\frac{\cancel{10 \text{ MPa}\sqrt{m}}}{1.12\cdot 10\cancel{0 \text{ MPa}\sqrt{\pi\times.001m}}}\right)^4 \frac{2}{2}\left[1-\left(\frac{1}{165}\right)\right]$$

$$= 10^6 \text{ cycles}\cdot\left(\frac{1}{3}\right)\left(\frac{1}{11.2\times.056}\right)^4\cdot\left[1-\frac{1}{165}\right]$$

$\approx 2.15\times10^6$ cycles. $\Leftarrow$ (15 points)

Thus, even if a sharp, 1 mm deep pre-crack *had* existed, under the [low!] design value of $\Delta\sigma_{des} = 100\ MPa$, it *should have taken* over 2 million load cycles to reach ~~a~~ a fracture length of $a_f = 165\ mm$. But failure occurred after only 140,000 loading cycles. This discrepancy is too large to accept, and ~~q~~ we would definitely NOT expect a failure to have occurred. We need to make another interpretation to square all of our facts an uncertainties.

5 pts

Extra Credit

Recall that there remains uncertainty in the actual value of σ_{min} in the cycle. Thus, there is also uncertainty in the actual value of $\Delta\sigma$ to be used in the fatigue propagation analysis.

Observe from the integrated, constant-Q and constant $\Delta\sigma$ equation, for $m \neq 2$, that with material properties, a_i, and a_f fixed, $N_{a_i \to a_f}$ is proportional to $(\Delta\sigma)^{-m}$

We can use this proportionality to infer an actual value for $\Delta\sigma$, based on our previous calculations and on the actual propagation life of $N_{a_i \to a_f} = 140{,}000$.

4.5

proportionality constant cancels

$$\frac{N^{(actual)}_{a_i \to a_f}}{N^{(pred;\,\Delta\sigma=\Delta\sigma_{des})}_{a_i \to a_f}} = \frac{\cancel{C}/(\Delta\sigma_{act})^4}{\cancel{C}/(\Delta\sigma_{des})^4}$$

$$\frac{140{,}000}{2.1 \times 10^6} = \left(\frac{\Delta\sigma_{des}}{\Delta\sigma_{act}}\right)^4$$

(8)

10 points $\Rightarrow \Delta\sigma_{act} = \Delta\sigma_{(des)} \cdot \left(\frac{2.15 \times 10^6}{140{,}000}\right)^{1/4}$

$$= 100\,\text{MPa} \cdot (15.36)^{1/4}$$

(5) $\boxed{\Rightarrow \Delta\sigma_{act} = 197\text{ MPa}}$

Thus, we infer an actual value of $\Delta\sigma \simeq 200$ MPa so that, with $\sigma_{max} = 200$ MPa, it would appear that the actual value for σ_{min} was $\simeq 0$ so that $\Delta\sigma \doteq 200\text{ MPa} = \sigma_{max} - \sigma_{min} \doteq 200\text{ MPa} - \sigma_{min}$

(5) $\Rightarrow \sigma_{min} \simeq 0$!

With this revised [enlarged] estimate of actual σ_{min}, the observed fatigue crack propagation life of only 140,000 cycles is easily understood.