LECTURE 9

Hydraulic machines III and EM machines

2.000 DC Permanent magnet electric motors

Topics of today's lecture:

- **Project I schedule revisions**
- **Test**
- **Bernoulli's equation**
- **Electric motors**
  - ✓ Review I x B
  - ✓ Electric motor contest rules (optional contest)
- **Class evaluations**

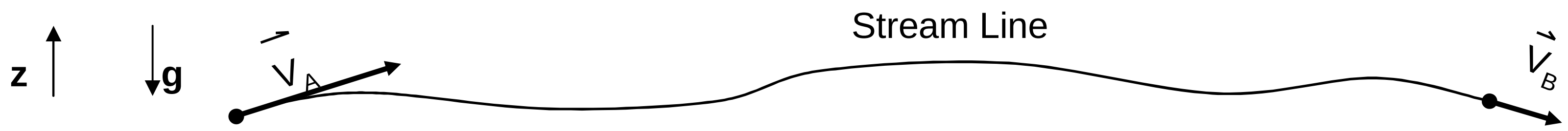
Project schedule updates

Approx START	WHAT	DUE	PTS
07 March	Project mgmt spread sheet	14 March	[20]
12 March	HMK 6: 1 page concept & equations + SIMPLE 1 page explanation	02 April	[80]
19 March	Gear characteristics 1 page explanation	02 April	[10]
19 March	CAD files & DXF files	09 April (via zip disk)	[90]
		Σ:	200

BERNOULLI'S EQUATION

Streamlines

Streamline: Line which is everywhere tangent to a fluid particle's velocity.



For a steady flow, stream lines do not move/change

A stream line is the path along which a fluid particle travels during steady flow.

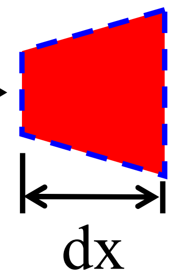
For one dimensional flow, we can assume that pressure (p) and velocity (v) have the same value for all stream lines passing through a given cross section

Bernoulli's equation for steady flow, constant density: $\frac{v^2}{2} + \frac{1}{\rho} \cdot p \cdot + g \cdot z = \text{Constant}$

Bernoulli derivation

For two cross section (ends of control volume) located dx apart:

$Something_{in}$ →



→ $$Something_{out} = Something_{in} + \frac{d(Something)}{dx} dx$$

$$v_{out=}v_{in} + \frac{dv}{dx} dx$$

$$A_{out=}A_{in} + \frac{dA}{dx} dx$$

$$p_{out=}p_{in} + \frac{dp}{dx} dx$$

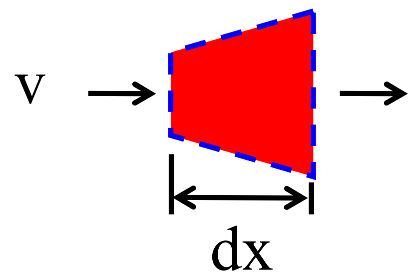
$$z_{out=}z_{in} + \frac{dz}{dx} dx$$

→ Differntial changes with dx

Bernoulli derivation

Stead flow momentum equation for Control Volume

From F = m a following a fluid mass



$$v + \frac{dv}{dx}dx$$

$$\dot{m}_{in} \cdot v_{in} + \Sigma F_{\text{on CV}} = \dot{m}_{out} \cdot v_{out}$$

For a stead, there is no stored mass

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$\dot{m} \cdot v_{in} + \Sigma F_{\text{on CV}} = \dot{m} \cdot v_{out}$$

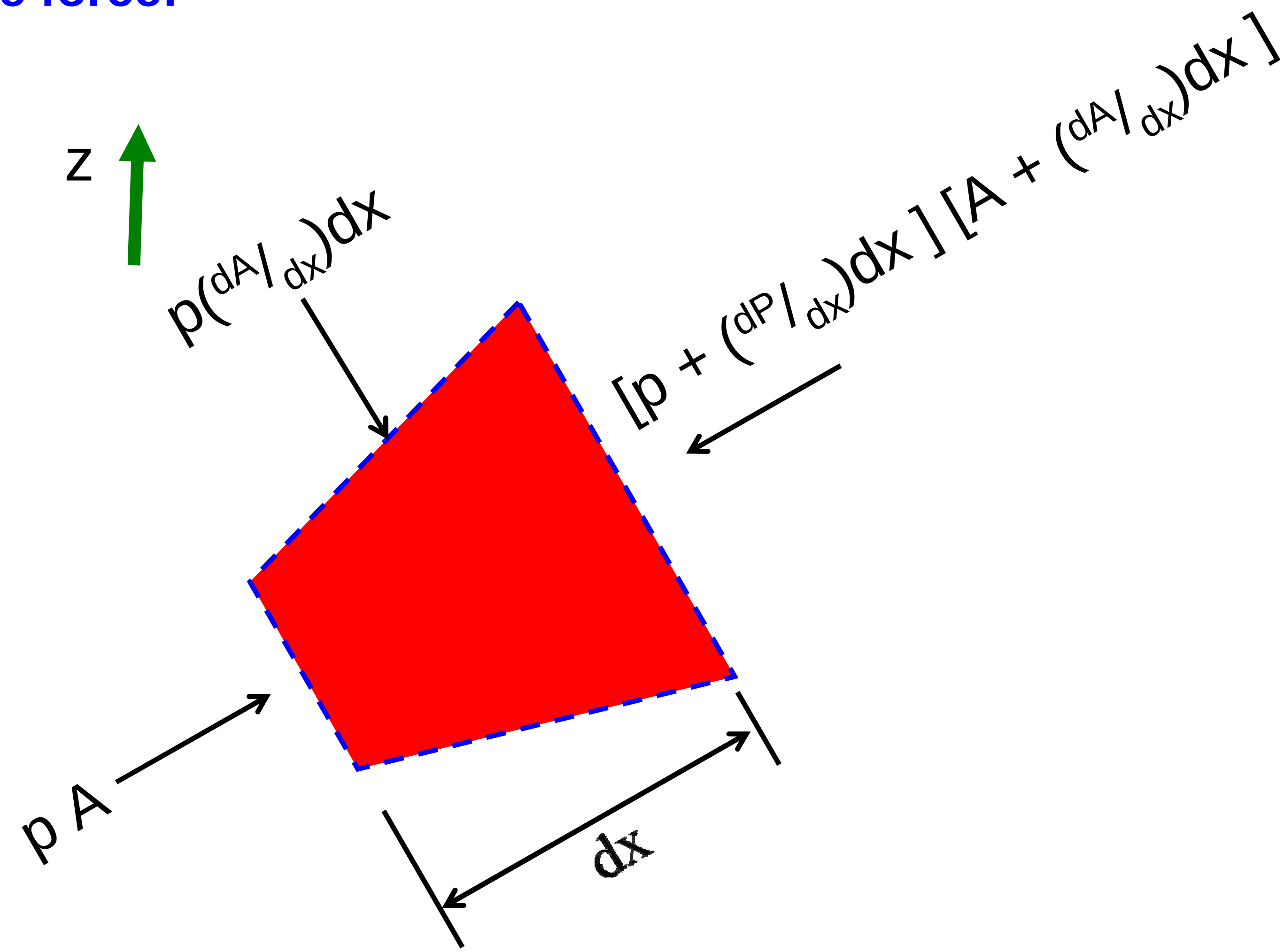
$$\Sigma F_{\text{on CV}} = \dot{m} \cdot \left(v_{out} - v_{in}\right)$$

$$v_{out} = v_{in} + \frac{dv}{dx}dx$$

$$\Sigma F_{\text{on CV}} = \rho \cdot A \cdot v \cdot \left(\frac{dv}{dx}dx\right)$$

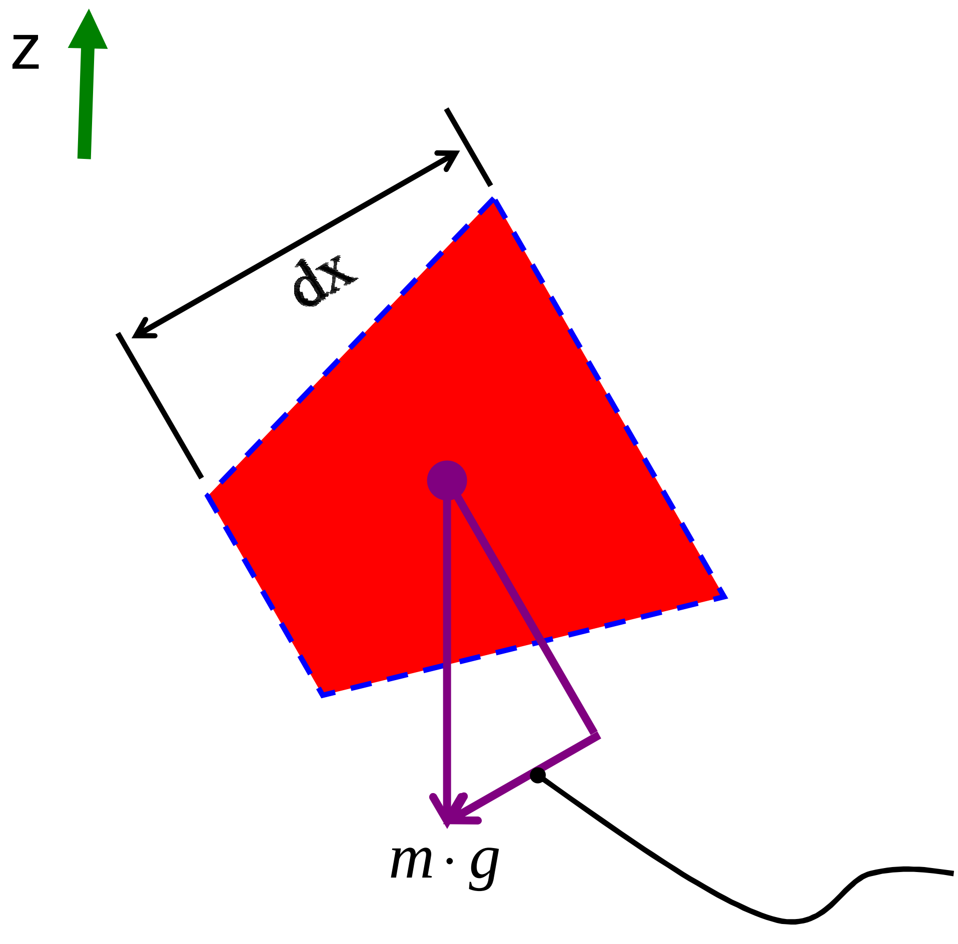
Bernoulli derivation

Pressure force:



Bernoulli derivation

Gravity:



$$m \cdot g \cdot \left(\frac{dz}{dx} \right) = -F_x$$

Bernoulli derivation

Summation of pressure and gravity forces:

$$\Sigma F_{\text{on CV}} = p \cdot A + p \cdot \left(\frac{dA}{dx} dx\right) - p \cdot A - p \cdot \left(\frac{dA}{dx} dx\right) - A \cdot \left(\frac{dp}{dx} dx\right) - (\text{terms}) \cdot dx^2 - m \cdot g \cdot \left(\frac{dz}{dx}\right)$$

$(\text{terms}) \cdot dx^2 \sim 0$

$$\Sigma F_{\text{on CV}} = -A \cdot \left(\frac{dp}{dx} dx\right) - m \cdot g \cdot \left(\frac{dz}{dx}\right)$$

$$m = \rho \cdot A \cdot dx$$

$$\boxed{\Sigma F_{\text{on CV}} = -A \cdot \left(\frac{dp}{dx} dx\right) - \rho \cdot A \cdot g \cdot \left(\frac{dz}{dx}\right) \cdot dx}$$

Bernoulli derivation

Equating momentum flow and applied forces:

$$\Sigma F_{\text{on CV}} = -A \cdot \left(\frac{dp}{dx} dx \right) - \rho \cdot A \cdot g \cdot \left(\frac{dz}{dx} \right) \cdot dx \quad = \quad \Sigma F_{\text{on CV}} = \rho \cdot A \cdot v \cdot \left(\frac{dv}{dx} dx \right)$$

$$-A \cdot \left(\frac{dp}{dx} dx \right) - \rho \cdot A \cdot g \cdot \left(\frac{dz}{dx} \right) \cdot dx = \rho \cdot A \cdot v \cdot \left(\frac{dv}{dx} dx \right)$$

$$-\cancel{A} \cdot (dp) - \rho \cdot \cancel{A} \cdot g \cdot (dz) = \rho \cdot \cancel{A} \cdot v \cdot (dv)$$

Exact differentials for constant ρ

$$-\left(\frac{dp}{\rho} \right) - g \cdot (dz) = v \cdot (dv) \longrightarrow \frac{v^2}{2} + \frac{1}{\rho} \cdot p + g \cdot z = \text{Constant}$$

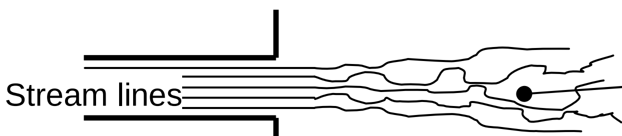
Bernoulli's Equation: Assumptions

Flow along streamline

- **B.E. can only be used between points on the SAME streamline.**

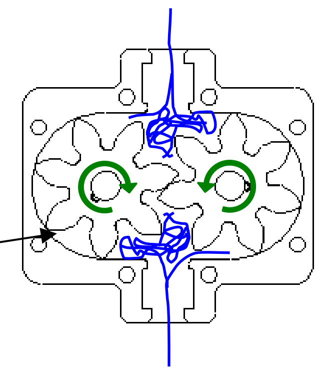
Inviscid flow:

- **Loss due to viscous effects is negligible compared to the magnitudes of the other terms in Bernoulli's equation.**
- **Bernoulli's equation can't be used through regions where fluids mix:**
  - ✓ Mixed jets & wakes (flow want to break up, swirl… resulting shear dissipates energy)
  - ✓ Pumps & motors
  - ✓ Other areas where the fluid is turbulent or mixing.



Mixed Jet

Mixing = Can't Use Bernoulli's Equation
You can not use Bernoulli's Equation
through jets or turbulent areas



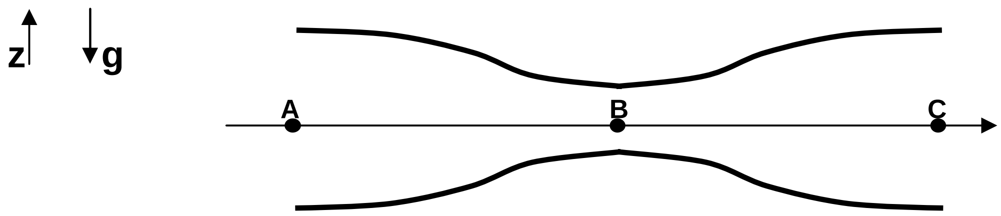
Stead state

- **Velocity of the flow is not a function of time, BUT!!! it can be a function of position.**

Incompressible

Bernoulli's Example – Pipe of variable diameter

Given v_A, find the pressure difference between A & B as a function of A_A, A_B, and v_A. Is this a rise or drop in pressure?



✓ Assumptions (along streamline, inviscid, stead state, incompressible)

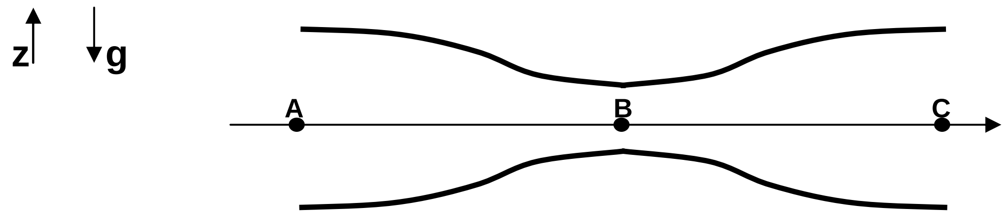
Bernoulli's equation between points A and B:

$$\frac{v_A^{\;2}}{2} + \frac{1}{\rho} \cdot p_A \cdot + g \cdot z_A = \frac{v_B^{\;2}}{2} + \frac{1}{\rho} \cdot p_B \cdot + g \cdot z_B$$

$$\text{Note: } z_B = z_A$$

$$\frac{v_A^{\;2}}{2} + \frac{1}{\rho} \cdot p_A = \frac{v_B^{\;2}}{2} + \frac{1}{\rho} \cdot p_B \longrightarrow \left(p_B - p_A\right) = \frac{\rho}{2} \cdot \left(v_A^{\;2} - v_B^{\;2}\right)$$

Bernoulli's Example – Pipe of variable diameter



$$\left(p_B - p_A\right) = \frac{\rho}{2}\cdot\left(v_A{}^2 - v_B{}^2\right)$$

Volume flow rate equality:

$$Q_A = Q_B = Q_C \quad \text{so} \quad A_A \cdot v_A = A_B \cdot v_B = A_C \cdot v_C$$

$$v_B{}^2 = v_A{}^2 \cdot \left(\frac{A_A}{A_B}\right)^2$$

$$\left(p_B - p_A\right) = \frac{\rho}{2}\cdot\left(v_A{}^2 - v_A{}^2 \cdot \left(\frac{A_A}{A_B}\right)^2\right) = \boxed{\frac{\rho}{2}\cdot v_A{}^2 \cdot \left(1 - \left(\frac{A_A}{A_B}\right)^2\right)}$$

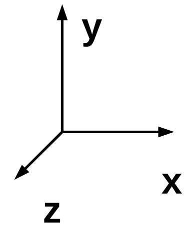
$$\frac{A_A}{A_B} > 1 \quad \text{so} \quad 1 - \left(\frac{A_A}{A_B}\right)^2 < 1 \quad \text{so the pressure drops!}$$

DC PERMANENT MAGNET MOTOR

Vector cross product review

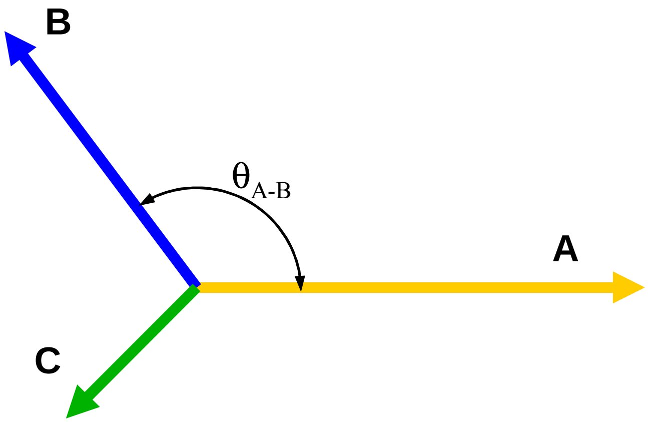
Vector cross products:

$$\vec{A} \times \vec{B} = \vec{C}$$



Mutual perpendicularity:

$\vec{C}$ is mutually $\perp$ to $\vec{A}$ and $\vec{B}$



Magnitude:

$$\left|\vec{C}\right| = \left|\vec{A}\right| \cdot \left|\vec{B}\right| \cdot \sin(\theta_{\text{A-B}})$$

When:

- **A & B are PARALLEL, the magnitude of (A X B) is 0**

$$\theta_{\text{A-B}} = 0^{o} \rightarrow \left|\vec{C}\right| = \left|\vec{A}\right| \cdot \left|\vec{B}\right| \cdot \sin\left(0^{o}\right) = 0$$

- **A & B are PERPENDICULAR, the magnitude (A X B) is maximized**

$$\theta_{\text{A-B}} = 90^{o} \rightarrow \left|\vec{C}\right| = \left|\vec{A}\right| \cdot \left|\vec{B}\right| \cdot \sin\left(90^{o}\right) = \left|\vec{A}\right| \cdot \left|\vec{B}\right|$$

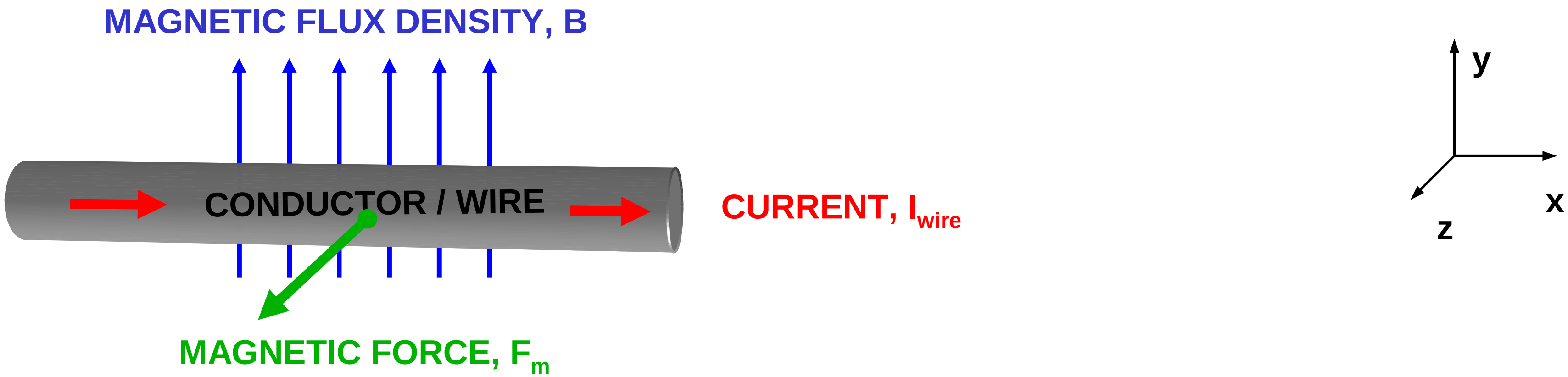
Magnetic force on a conductor (wire)

Force on a conductor carrying a current through a magnetic field:

$$\vec{F}_m = \vec{I} \times \vec{B}$$

Where:

$\vec{F}_m =$	Magnetic Force	$[\text{N or lbf}]$
$\vec{I}_{wire} =$	Current	$\left[\text{Amps or C}\,\frac{\text{m}}{\text{s}}\right]$
$\vec{B} =$	Magnetic flux density	$\left[\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}} \text{ or } \frac{\text{Weber}}{\text{m}^2}\right]$



Magnetic torque on a simple electric machine

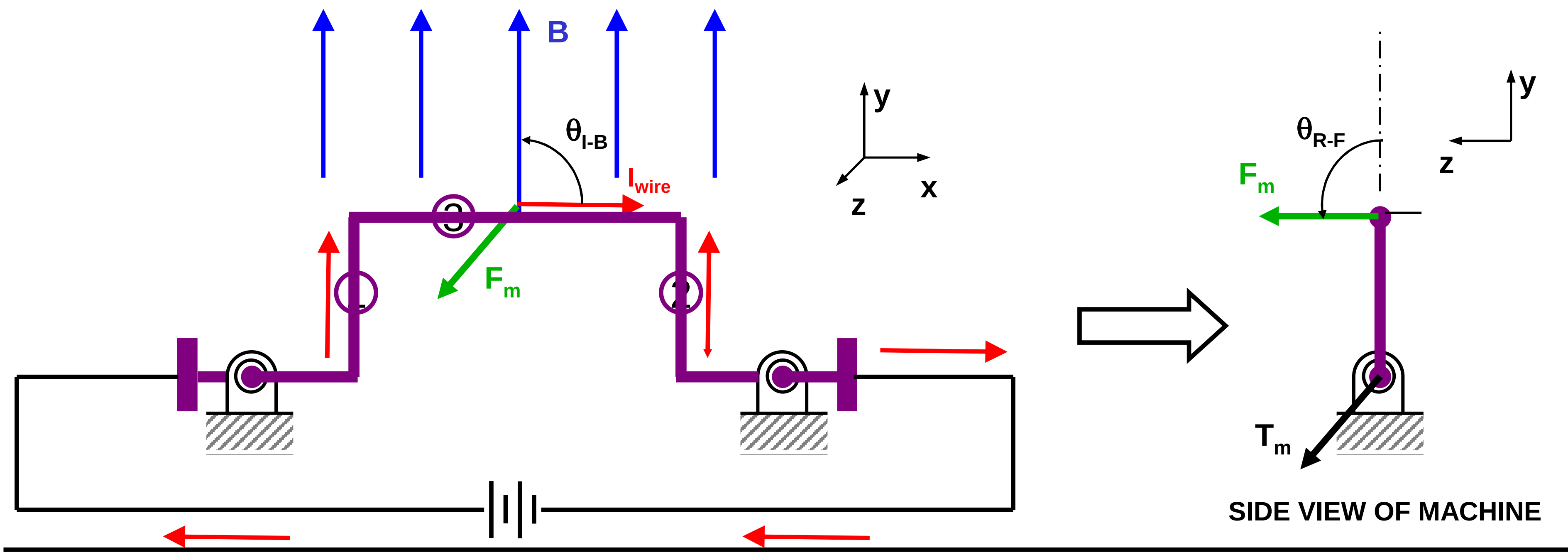
Force on a conductor carrying a current through a magnetic field:

For wire 3, θ_{I-B} always = 90º so $\sin(\theta_{I-B})$ always = 1

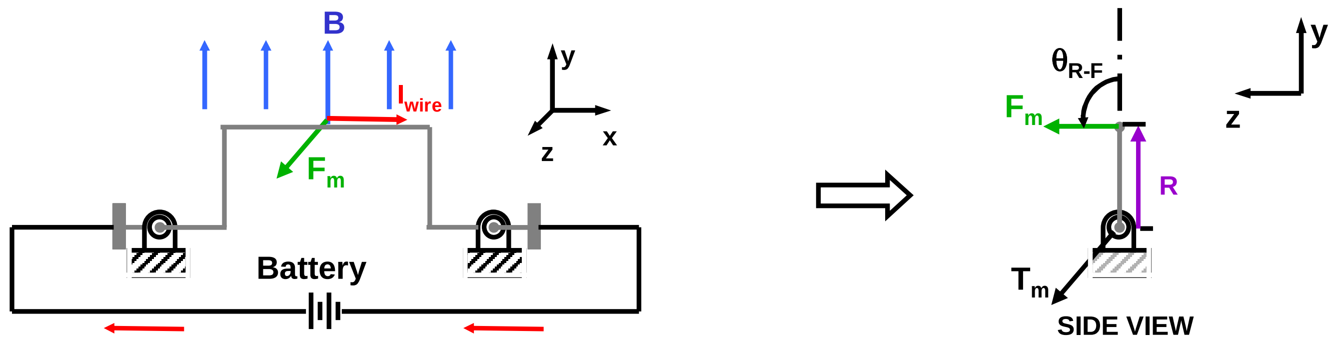
Force on wires 1 (to left) and 2 (to right) do not act to make wire rotate

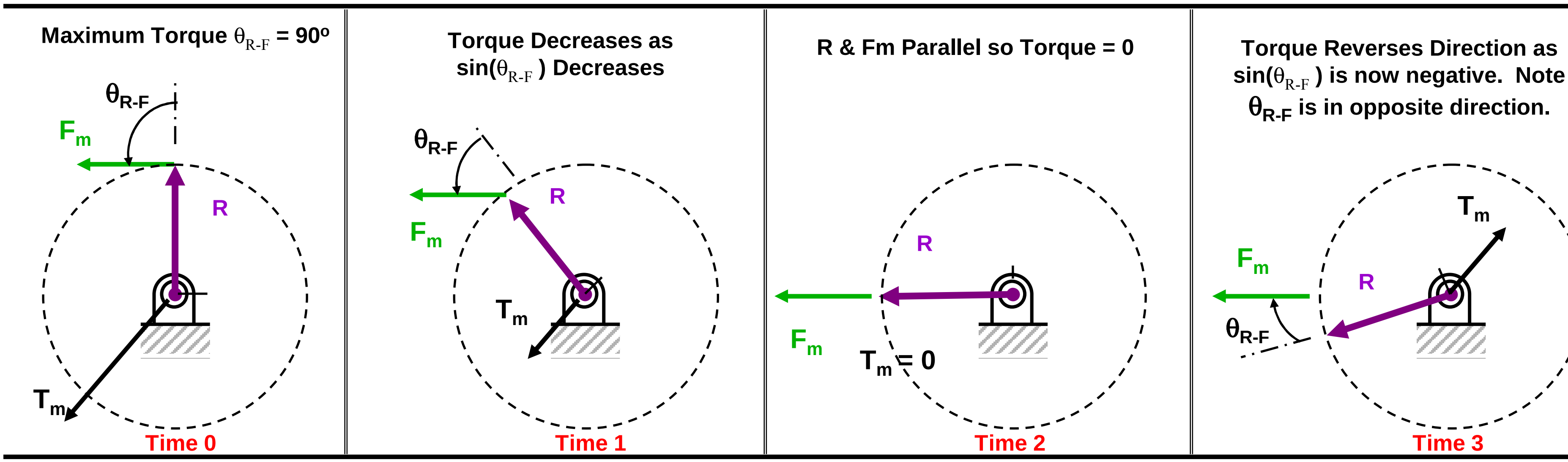
$$\vec{F}_M = \vec{I} \times \vec{B} \longrightarrow \left|\vec{F}_M\right| = \left|\vec{I}\right| \cdot \left|\vec{B}\right| \cdot \sin(\theta_{I-B}) = \left|\vec{I}\right| \cdot \left|\vec{B}\right|$$

$$\vec{T} = \vec{r} \times \vec{F} = \vec{r} \times (\vec{I} \times \vec{B}) \longrightarrow \vec{T} = \left|\vec{r}\right| \cdot \left|\vec{I}\right| \cdot \left|\vec{B}\right| \cdot \sin(\theta_{R-F})$$

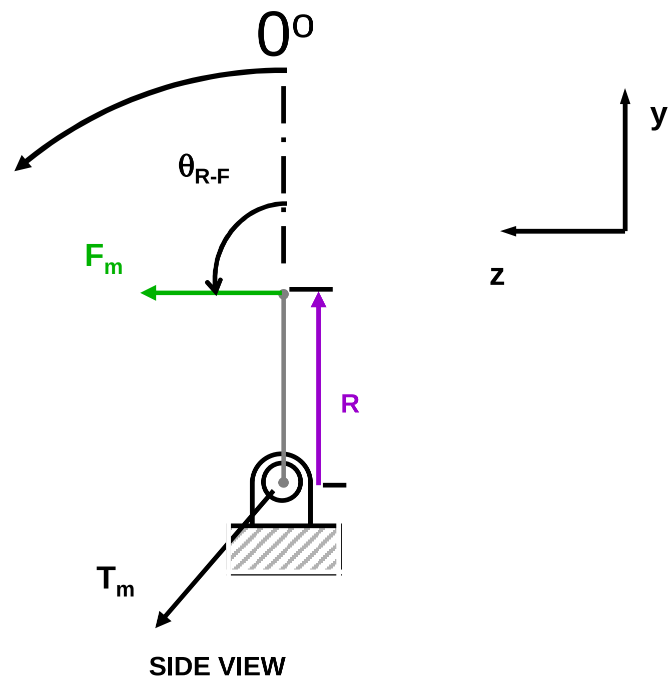


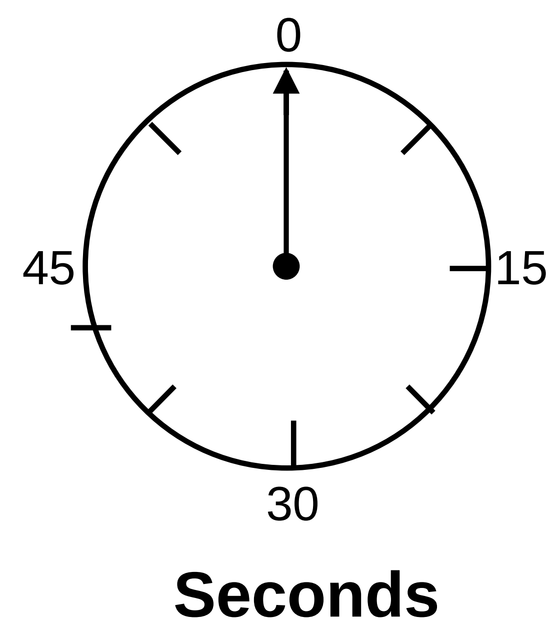
Magnetic torque as a function of position



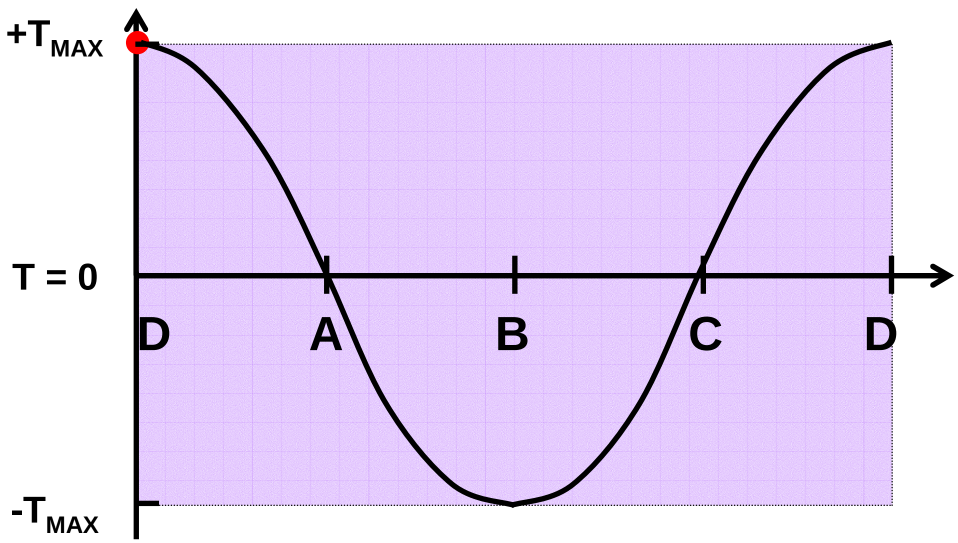


What does the torque vs θ curve look like? [2 mins]

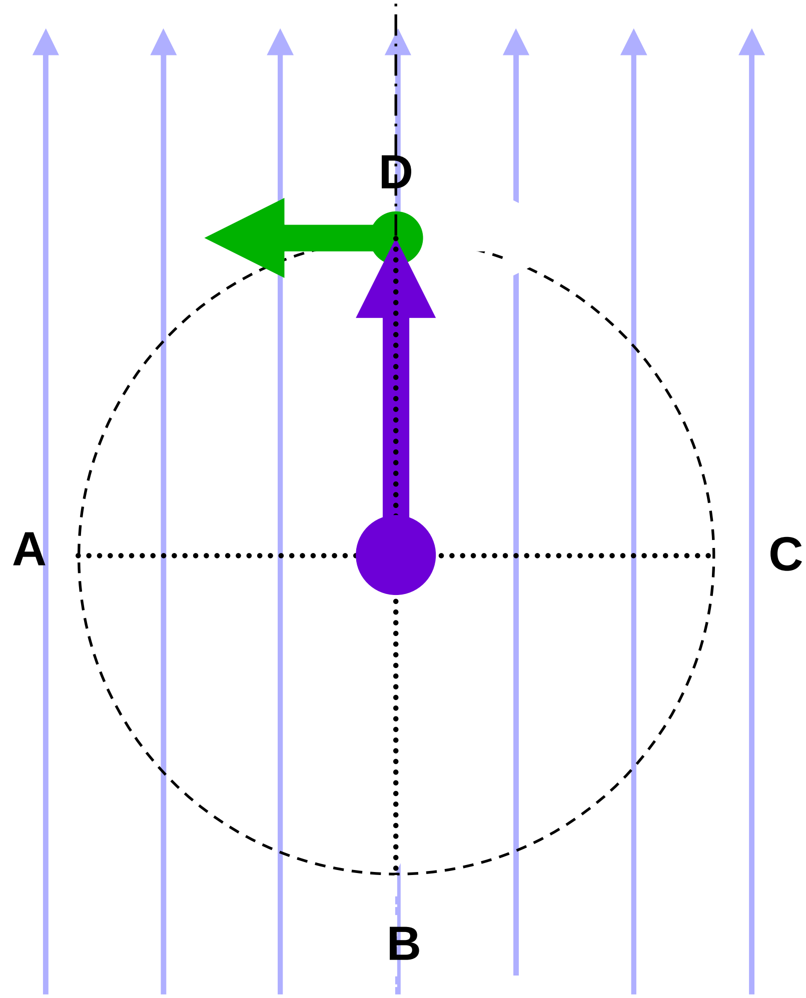




Torque on simple wire loop carrying current



Torque curve of simple loop



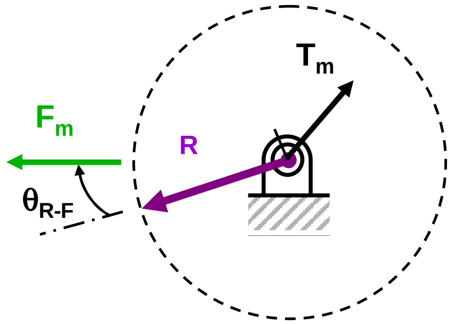
Side view of simple loop

Keeping the machine in motion

How to keep the machine moving

- Once the wire passes horizontal, T_m tries to stop the wire from rotating.
- To keep the wire rotating, we must either shut off the current or reverse the current.
- If we turn off the current, the wire will continue to rotate due to its inertia.
- If we reverse the current direction when the wire reaches horizontal, T_m will act to keep the wire spinning

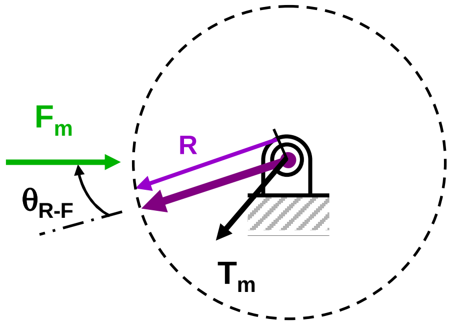
If current continues in the same direction, Tm tries to stop wire from spinning.



At time 3 Without Current Switch

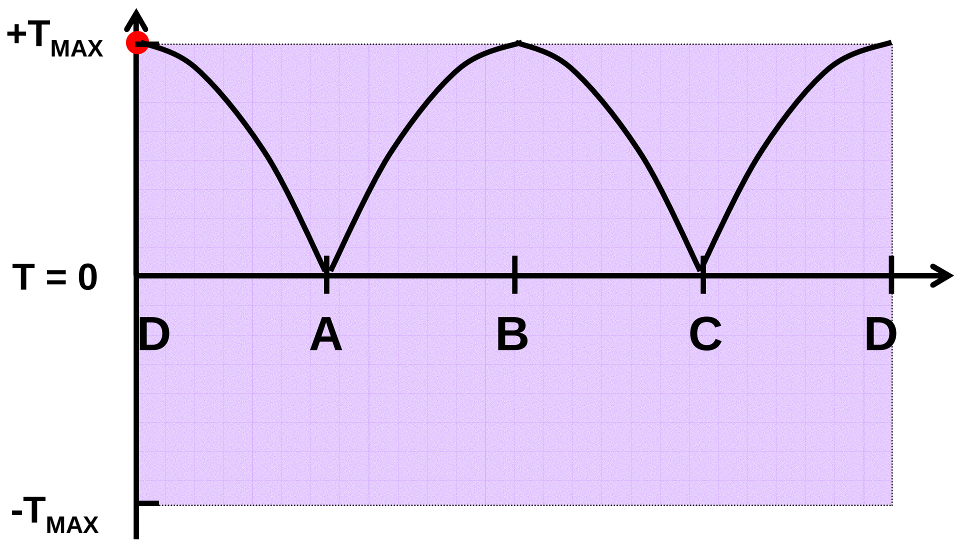
Changing current direction will change the direction of Fm.

This in turn switches the direction of Tm. Tm will now act to keep the wire spinning.

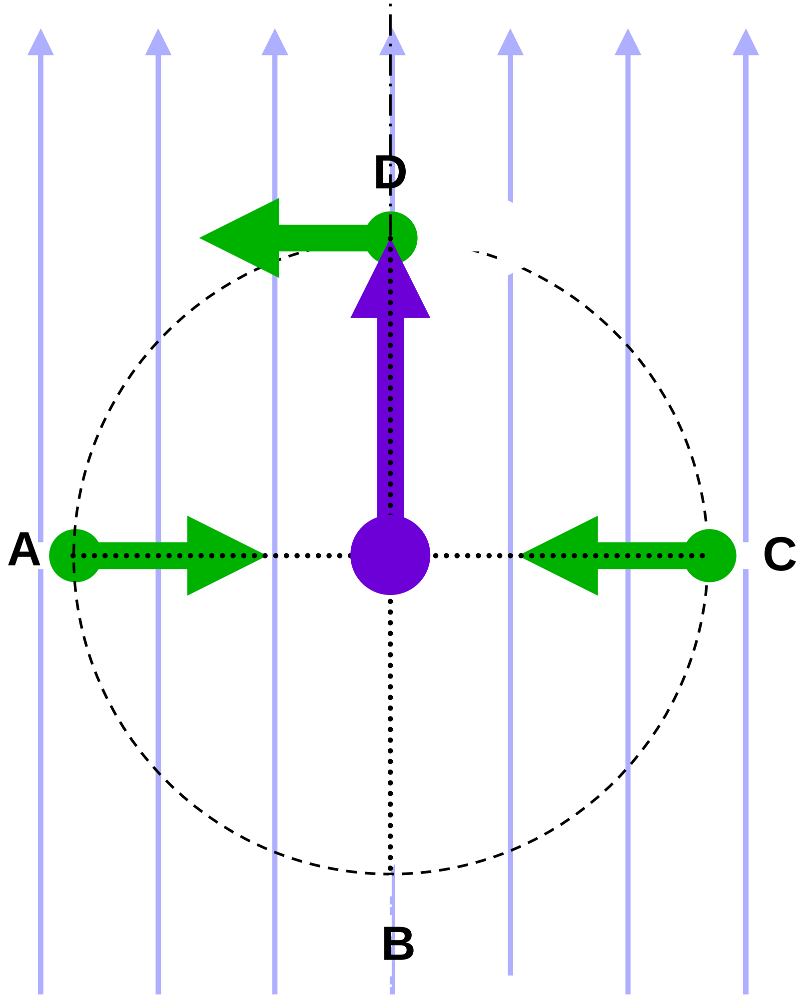


At time 3 With Current Switch

Torque on switched wire loop carrying current



Torque curve of switched loop



Side view of switched loop

DC Permanent magnet electric motor build & contest

In your kit you will find materials to build a simple electric motor

How it works:

- **Motor current shuts off when torque becomes negative**
- **Rotor inertia carries rotor until current turn on**
- **Repeated cycle keeps the motor spinning.**

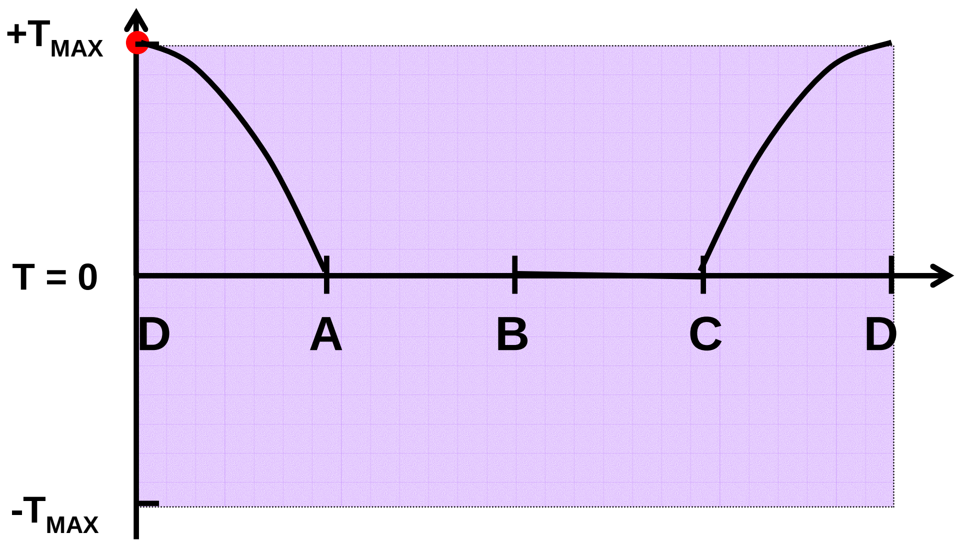
We will have a contest (in your lab sessions) to determine winner

- **How do you maximize energy input?**
- **How do you minimize losses?**
  - ✓ Friction
- **You may need to try various things**
- **Class record = 1800 RPM!**

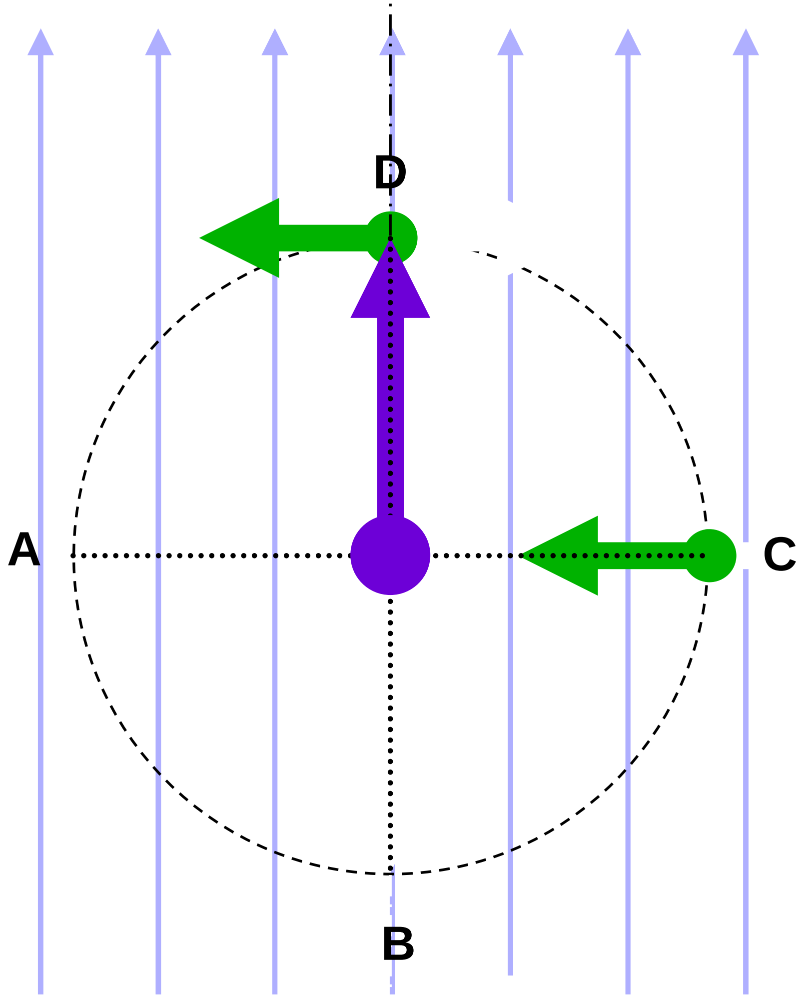
Prizes

- **Fastest motor = $20 Cheesecake Factory gift certificate**
- **Record breaker = Keep Lego kit at end of class**

Torque on switched wire loop carrying current



Torque curve of switched loop



Side view of switched loop

DC Permanent magnet electric motor build & contest

Contest rules:

- **The contest is OPTIONAL**
- **Motor may only contain the materials in your kit and a roll of life savers**
- **You may use the contents of your tool kits to help shape/make the motor**
- **You may not use any other tools/machines to make the motor**
- **Any wire coil must be wound around the battery or the roll of life savers**
- **You may obtain up to 3 ft of additional wire from a TA if you need it**
- **Your motor may only be powered by our power source (fresh D battery)**
- **We will test them in lab next week**