PROFESSOR: Anyway, welcome today. Stefan Andreev is our guest speaker from Morgan Stanley. And as I understand you have a degree, a PhD Degree in chemical physics.

STEFAN ANDREEV: In chemical physics, yes. And maybe I should go here.

[LAUGHTER]

PROFESSOR: And now he's in the world of finance. And we're here to benefit from your experience.

STEFAN ANDREEV: Thank you very much for the introduction. Yeah. I went to school at Dartmouth College undergrad, and then up the street at Harvard for my PhD. And then I transitioned from science to finance. And for the last eight years, I've been working at Morgan Stanley, working with [INAUDIBLE], an instructor in the course.

So today, what are we going to be talking about? Well, to give you a big picture view of where our topic fits within the grand scheme of finance-- in general, there are really two big areas in my view. Well, there's probably more, but these are kind of most famous, I would say, areas where quantitative skills can be-- are very valuable in finance.

And the two areas are-- one area is statistics predictions, which is essentially, say given some historical behavior in the market, how do we predict what will happen in the future? And that's certainly a huge industry. People have made a ton of money applying quantitative concepts to that. But that's what we're going to talk about today.

What we're going to talk about is another very big area called pricing, which is pricing and hedging of complex instruments. And that area is really about essentially when you have a complex product that you don't really know the price of, but you know the prices of other products. And then you can use the other products to essentially replicate the payoff of your complex product.

Then you can use mathematical techniques to essentially say, look, the main statement is hey, because I can replicate my payoff, and using products that I know the price of, then that means that I can say something about the price of my complex product. I can basically price it. And not only can I price it, but I can also-- when I give the price, I know that I can eliminate any uncertainty from owning the product by executing a hedging replication strategy-- at least theoretically speaking.

So that's the area we're going to focus today. And our main focus is going to be on FX-- foreign exchange-- interest rates, and credit-- and in particular about credit FX hybrid models. We're going to be talking about essentially what happens, why do we need credit FX hybrid models, and going through an example of a simple one, and how to apply it.

In particular-- and there's the mathematical techniques we're going to be using-- as I said, we are going to be talking about the risk neutral pricing, which is essentially replication. And we're going to talk about how to use jump processes-- which you might have seen in other parts of your studies as Poisson processes-- to describe certain behaviors of price behavior that you cannot really describe very easily using pure diffusion Brownian motions that you probably have seen so far in the course.

And why do we care about that? Well, there are certain financial applications where this is important. And in particular, something that happened in the last few years-- the sovereign crisis in Europe.

And also, it has happen not just last year. This happened many times in other parts of the emerging markets. And given the emerging markets as my background, I've worked on these kind of models.

And this is, when you have Greek bonds in Euros, and there's a potential for Greek default. And as we know, as you might have read in the news, there was really a big worry about what will happen to the Euro currency if there is a spate of sovereign defaults.

And in fact, Euro currency did-- in anticipation of the possibility of default-- it actually did depreciate for while back in 2011 and 2012. Now it's pretty much back where it was before that, but it certainly-- there was a fear in the market-- which was also very, very obvious in terms of option prices-- that Euro currency could depreciate significantly if, in fact, a disorderly default did happen.

Now it didn't happen, so that's good. But in other emerging markets in history, it has happened before. So it's not really an empty question.

So foreign exchange-- how do we describe it in math finance? Well, we think of it as the price of a units of foreign currency in dollars. In our presentation we're going to denote the spot FX rate, which is the current rate of exchange, by S. And here is a sample graph of your USD FX rates. You can see it looks like a random walk. It's very well described in normal circumstances as random walk.

So one very fundamental property that connects FX and interest rates is the so-called FX forwards interest rate parity, which says if I have a certain amount of money-- in this example, $5 million, and I can invest it, there's two ways I can utilize this money. One way is to just invest it at a dollar kind of risk-free rate. And we're assuming here we have a risk-free rate-- the standard assumption.

Or we can do something like we can take the money, exchange it into, say, Euros, invest it using the Euro risk-free rate, and then exchange it back into dollars. And this is essentially used to price of FX forward contracts. So FX forward contracts are a contract that allow you to say, look, I'm going to agree with you. Then in one month's time, I'm going to, say, give you 4,108,405 Euros, and you're going to give me back $5,170,000.

It's essentially an agreement that's a derivative contract. And if you see the-- if you have this forward contract, you can lock in, essentially, through conversion in Euros.

So you can lock in an effective dollar interest rate.

So FX forwards can be essentially described fully by knowing the interest rates in each currency and the spot FX rate. Conversely, you can infer foreign interest rates knowing the FX forwards. They're very connected.

Yes?

AUDIENCE: In this example, there's no mis-pricing, so you get that same amount. Is that the idea?

STEFAN ANDREEV: In this example, there is no mis-pricing. You get back the same amount. So we are assuming, essentially, there's no arbitrage.

We're not assuming, but we're given-- if the prices were, indeed, if this interest rate-- 4.6% in Euros-- interest rate was 4.6, in dollars it's 2.4. And here are the current spots, which is 127 and the forward, 125-- if these were, in fact, the observable mark quantities in the market, then there would be no arbitrage. And there is-- you're basically indifferent whether you invest the money in dollars, or you go the way of exchanging into Euros, and investing in Euros, and then back into dollars.

So in this example, the way I've presented, worked it out, there is no arbitrage. Now, if some of these numbers-- say the interest rate in Euros were 4% instead of 4.6, and all the other quantities were the same, then, in fact, there would be arbitrage. And you could make money by borrowing money in dollars and investing. I mean, the purpose of this slide is really to illustrate kind of hey, if there's no arbitrage, how one would actually compare-- how one would actually look for arbitrage in this example. This is-- again, this is a little bit of definition what are compound interest, interest rates.

We're going to talk about instantaneous risk-free rates. We're going to, again, say they're risk-free. So basically, we know for sure we're going to get our money back. You can think of risk-free rates as the one that treasuries pay in real life, or the one that Federal Reserve guarantees on deposits.

There's various examples of risk-free rates. And while, in practice, different risk-free rates can actually be different, so they're not really risk-free. But in our world right now, in our model, we're going to assume that there is such a thing as a risk-free rate for every currency, and it's unique.

And now, as we talk about our dynamics of the FX process, what's really focused on here is an effect. We're making an FX model. And we want to see-- in the previous example, we saw if you're given a FX rate and given some interest rates, here's what the FX forward really has to be in order to have no arbitrage.

Well, now we are trying to describe. We tried to describe or define a process for the FX currency. Essentially, this kind of no-arbitrage condition leads to having certain constraints on what the stochastic differential equation has to be.

So in this particular case, the constraint is that the drifts of the process has to be the differential in interest rates. So if one currency pays more than the other currency, obviously, people would want to invest in that currency. So that in order for no arbitrage to exist, there has to be an expectation that the currency that pays more would depreciate in the future, otherwise it would be an arbitrage.

So if it doesn't depreciate, if you can kind of say, hey, this currency won't depreciate, then you can just always invest in that currency that pays higher interest rates and make money-- which, in fact, many people do. But again, that's a-- they're taking a certain risk. They're taking the risk that the currency will depreciate.

So what do we actually want? What we want is to say-- we want to essentially-- and for the arbitrage conditions from before-- which is to say that my forward rate has to be essentially the spot rate-- well, this condition here has to be observed, essentially. And what does that mean? It means that my forward rate has to be my spot--

AUDIENCE: [INAUDIBLE] each other, you mean?

STEFAN ANDREEV: What'd you say?

AUDIENCE: [INAUDIBLE] set those equal to each other?

STEFAN ANDREEV: Yes. My forward rate has to equal to the spot rate times, essentially, the interest rate differential. If that is true, than in the previous set up, there will be no arbitrage. And why is that? Well, that's because the amount of money I earn on the domestic leg is e to the rd. The amount of money I earn on the foreign leg is e to the rf, but then I multiply by the forward, and that has to equal to the e to the rd.

So this is a standard. This is a the most basic dynamic FX model that people use in industry. It's referred to as the Black-Scholes FX model. And the stock price-- you've seen stochastic models before.

Usually, you see the stock price when people talk about options. In that case, this drift is just the risk-free interest rate. Well, here in FX, it's a differential of interest rates. Otherwise, it's very similar.

So FX has some interesting properties. So we're got talk the game. And before we go to the game, one question-- can FX exchange rate ever be negative?

What do you guys think? Can the dollar-Euro exchange rate be negative? Any ideas?

No, it's hard, because what does negative mean? It means I have to pay you money to give you Euros. Why would you? You have to pay me money to give me Euros. Nobody would do that. It can be 0, potentially, if dollars are worthless or something, but it cannot really be negative.

So that's one reason why I wrote my SDE as a kind of a log-normal process. You recognize this by the form dS over S. So the changes in the FX are proportional to the value of the FX. So the process can never become negative.

OK, so it can never become negative, but how big can it get? And the answer is, it can get very big. I mean, we have currencies-- not to blame some currency like Zimbabwean dollars, that traded-- I don't know, I mean, I actually don't know what

Zimbabwean dollars trade at, but I think it's somewhere in the billions of Zimbabwean dollars per dollar, so something. It's a really extreme example. It can really get extremely big.

So there is now really upper bound, while there is a lower bound. So the distribution, as you can imagine has a skew. It's not symmetric around the average. It's limited on the lower side. It can go very high on the high side.

And log-normal distribution has that property. Have you guys seen a log-normal distribution? You've talked about this stuff in the course before, right?

So let's go back to our game. So our game is, we have assumptions. My assumptions are not to be realistic, but to make it simple. Let's assume that our dollars and Euros exchange rate is one, so we can exchange one Euro for $1. Clearly not exactly the case, but let's make that assumption.

And we also assume that the FX forwards is 1, which basically means that the interest rates in both currencies are the same. And now let's say I'm going to make you a bet that-- now dollars and Euros is a volatile process. Right now, it's one, but in the future, it could be different from one-- could be higher or lower.

So if dollar a Euro FX process is more than one in one month, then you give me money. And then if it's less than one in one month, then I give you money. And we're going to have two payoffs, so two games.

I don't know why it says "bet beach." Should say just "bet." I'm sorry about that. That in the payoff A, basically you're going to give me $100 if I win, and I'm going to give you $100 if you win. And in payoff B, you're going to give me 100 Euros if I win, and I'll give you 100 Euros if you win.

And the question is, which game would you prefer to play, or do you not care? So in each case, you kind of win, lose same number. So I want to see hands. Who wants to play game A?

Come on guys, wake up. Who wants to play game A? I mean if you don't know, you just-- lets say you like Euros better. You can say this is not really graded, so it's OK.

OK, nobody really knows what to play? Like how about game B? Anybody wants to play game B? OK, you guys want to play? Three people for game-- four-- game A nobody still? Same person for game A and B? All right.

OK, two people-- so now--

AUDIENCE: Behavioral science says people are reluctant to lose-- more reluctant to lose.

STEFAN ANDREEV: That's true, that is true. However, so people are reluctant to lose. And I said, look, the FX forward in one month is 1. So you can actually-- that's the market price that in one month, FX forward is 1. And this-- our bet-- kind of the strike is 1. So our bet level is 1.

So you can kind of say, well, this looks like kind of fair game. So I don't expect to win or lose much, but I'm just reluctant to do it. And I can get that feeling. That's the risk aversion aspect of it.

But if you were forced to make a bet, question is, which one would you prefer? So I understand you might not want to play. But I'll say, OK, so you guys don't seem to be in the mood to play. That's fine.

Let's look at some scenarios. So let's say in one month, dollar-Euro goes to 1.25. In bet A, I lose $100. In bet B, I lose 100 Euros. So bet A-- actually, you lose 100 Euros, not-- so bet A for you, you are $25 better than bet B.

And in the second case, if dollar-Euro is 0.75, you make $100, or you make 100 Euros in bet B. In that case, you also-- bet A is $25 better. So it doesn't matter what happens. Bet A seems to be the better case.

So if you're, like, our dear Professor here, then you don't like to lose, then you probably are going to choose Bet A, I assume, right? That's a better deal. And that's kind of strange, though.

I mean, like both payouts were symmetric-- so 100 Euros, 100 Euros, $100, $100.

Why is it one is better than the other? Well it's like what really happens is the units of the bet-- the value of those units-- depend on whether you win or lose. So it's not like if I was betting using acorns, then you get two acorns or I get two acorns, then actually, it might be a fair bet.

But because I'm betting in Euros and dollars, and the value of these things-- the relative value-- changes based on the actual whether you win or lose, then the game is not symmetric any more. So the reason I wanted to take you through this game is because there are a lot of cases in finance where people make bets. But then the value of what you get depends on whether you win or you lose. And that has an effect on the value of the bet.

And in particular, the case we're going to talk about today is one of these cases, which is the credit FX. That's why we need to create FX quantum models. To give you an illustration from finance, let's take Italy bonds.

So Italy issues bonds both in dollars and in Euros. Why does it issue in both currencies? Because Italy has to issue a lot of bonds. And they need to find as many investors as they can. And some investors want to buy Euro bonds, to and some investors want to buy dollar bonds. And they want to access both bases of investors.

Now, these bonds they cross-default, meaning if Italy defaults on one bond, all of its bonds default together, including the Euros and the dollar bonds. So then there is a notion of credit spread, which is the measure of how risky Italy is.

So you can take Euro bonds, and you can say, well, how much premium does Italy pay over German bonds? Let's assume that German bonds are risk-free, which is the standard assumption for Euros-- that German bonds-- Germany is the main underlying economic force for the Euro. They're kind of risk-free bonds. And Italy pays a certain spread over Euros. Same thing for dollars-- it pays a certain spread over USA.

So if Italy wants to borrow money, they have to pay a higher interest rate, just like if you want to borrow money for student loans, you have to pay higher interest rate than the Fed. And the size of that spread is in the market. It determines how risky of a borrower you are. Well, it turns out that these spreads are not the same in both currencies. One currency has a higher spread than other currencies.

That's kind of an interesting thing. So there are so two questions really-- when the spreads are not the same, which currency would Italy prefer to issue bonds in? And which currency do investors prefer to buy bonds in?

So this is kind of similar to the previous game we played, because if you're an investor trying to buy bonds, well, if Italy defaults, then chances are the Euro is not doing so well. So you would lose money. And if you have Euro bonds, you would lose Euros. If you have dollar bonds, you lose dollars.

On the other hand, if Italy does not default and pays you back, then chances are, Euros are not doing that bad. So you would actually be making Euros and dollars. So it's an interesting-- it's kind of a similar dynamic going on.

So there's the same kind of question that I asked before-- USD, Euros, or equal in both. So what do you think now? Now that we've gone through an example, maybe we'll have a higher participation in my pop quiz.

Who thinks that USD bonds have a higher credit spread, and who thinks-- so A. Vote for A. One, two. So who thinks that Euro bonds will have a higher credit spread? OK, one, all right-- so two to one. I think the two to one wins. All right, I must say, you guys seem that-- maybe it's the format of the auditorium. People don't like to raise their hands too much, or maybe they're afraid that they're being filmed. [LAUGHS]

OK. Well, how are we going to do this? How are we going to answer this question?

Before I give you the answer, we're going to go through a slide. Well, first we're going to say, well, FX rates are volatile. There is volatility, as we said before.

So now we're going to-- in order to compare Euro bonds to dollar bonds, we need to really come up with a strategy to replicate one with the other, and then look at the price-- look at how much do we need to buy one to replicate the other. If we're able to come up with such a replication strategy, then we can immediately say, hey, if you need 150 Euro bonds to replicate $100 bonds, then that means that the Euro bonds have to be cheaper. That's basically the replication argument.

So you can try to do that by piecing together bonds, or we can use the powerful tools of mathematical finance that you've been learning about, which is all about replication and pricing. And the three steps are-- we're going to analyze the payoffs of the instruments, and we're going to write some model-- a model for FX and for credit, and we're going to price those bonds. And then we're going to look at the results, and try to understand the problem intuitively.

And that's basically what we do, pretty much. That's what option quants do on Wall Street all the time. So here's the answer-- dollar versus Euro spreads from the marketplace.

So usually what happens in these kind of questions in finances, you kind of have an answer, and then you try to compute a model that explains the difference. So that's what we're going to do now. Well, the USD spreads are actually lower-- USD bond's spreads are actually lower.

Now, so there is really-- when we're talking about bonds, risky bonds, there's two states. They are either performing, or they are non-performing and in default. And we're going to go here through an example of two bonds. And we're going to use two zero coupon bonds, which essentially have zero recovery.

And the idea there is really to make the question simple so we can analyze it better. But you don't lose a lot of generality by saying zero coupon versus coupon. It's not the answer. The intuition would be exactly the same.

So let's say we have two zero coupon bonds, same maturity, they pay 100 on maturity. And by the way, bonds-- I don't know how much you guys have-- I say these things. I'm very familiar with them. Bonds are nothing more than loans.

So zero coupon bond means I give you some amount of money, and at some pre-agreed maturity, you're going to pay me 100. So let's say I give you $0.80, one year from now, you pay me 100. And I call this a zero coupon bond, because you don't pay me any intervening coupons. There's no interest payments, but just I pay you less money now, and you pay me more at maturity.

OK, so we know that bond U pays $100. Bond E pays 100 Euros. And let's say we denote the prices-- price of U is Pu, price of E is Pe. Our spot FX rate, we're are going to call it ST, our FX forward, FT.

Now we're going to have kind of a simple arbitrage strategy. Well, let's say if we can sell 100 times FT dollar bonds and with the proceeds, buy 100-- we're going to get-- if we sell 1,000 dollar bonds, we're going to get this much proceeds. There's the price.

And if we buy 1,000 Euro bonds, so we can enter into an FX forward contract for 100,000 Euros for maturity, P at zero cost. All right, so let's see how this strategy actually pays out. Well, what happens is you-- there's 100,000 Euros. You get 100,000 Euros for selling the Euro bonds.

You pay $100,000 times FT dollars, say dollar bonds. There is an FX forward contract, and at maturity, you can exchange this $100,000 for $100,000 Euros using this FX forward contract. You already agreed to do that.

So your FX forward actually exactly hedges. You can basically use the proceeds of these bonds to-- you can exchange the proceeds at zero cost at maturity, because you have entered into the FX forward contract. So your net payoff is 0.

So that means that the prices of these bonds have to be the same. But what if they're not? What if FT, which is in this case is one forward contract-- what if the price in dollars is different from exchange rate times the price in Euros?

Well, in that case, you can say, well, there is an arbitrage. And you'll be right, if you would be able to make money if, in fact, the bonds performed. But what happens if there is a default?

If there's default, that wouldn't really necessarily be the case, because if there's default, these bonds don't pay anything, and you just have an FX forward contract. And this FX forward contract is going to be worth something after default, especially if the FX rate depends, like jumps, upon default. So arbitrage, again, is so you start with 0 money. You make money if there's nonzero probability.

And let's say in this particular case, the strategy payoff in case of default with 25% recovery rate is-- you actually have only-- 25% means you only have a quarter of the payoff now at maturity, if default occurs. But you have a hedge for the full 100,000. Your FX forward is for full 100,000.

So for 25,000 of it, you can use the FX forward to exchange money. For the remaining 75,000 you just have an FX forward outright. So if FX moved against you, you would lose money.

So that's why the strategy is not necessarily an arbitrage. And that's why the two prices of the dollar and Euro bond are not necessarily related to each other. They don't have to be equal, because, in fact, there is a possibility of default.

And you cannot really directly hedge. You cannot really construct an arbitrage strategy by using FX forwards and the bonds together so easily. You have to take into account what happens if default occurs.

OK, so give an example. What happens with bond FX when default occurs? Well, one of the most recent defaults of a country-- of a big country that has its own currency-- is Argentina, 2001.

And when it defaulted, the Argentinian Peso skyrocketed. Here is the graph of the price series. So if you had an FX forward contract that essentially-- if you had a position where you were left with a naked FX forward contract, where you were receiving pesos and paying dollars in the event of default, you would have lost a lot of money when the default happened. It would have really gone against you.

And this is, by the way, this is a massive move. And the Argentinian peso still is not recovered from that default. So can we do better? What do we actually-- what should we be doing when we're hedging this?

And the answer is, again, we have to apply mathematical models to really try to come up with a replication strategy. So what is the main features of a model that will help me do this? Well, first I need to model a credit default, the credit default event. I need to have this in my model.

And I need to have something which says FX has to move upon default. And then we're going to construct a complete market. Then we're going to define some simple dynamics on our exchange rate and on our defaults, and we're going to try to price for bonds.

So how do we do that? Well, what we generally, again, how we're going to use the models, we're going to define an SDE, like I just defined initially a DS over S of something. And I'm going to solve this SDE using either analytically or numerically.

And what's important, the way we actually use these models in trading, we're going to look at how the price of each instrument depends on the hedging instrument. And that is going to define my hedge ratio or my replicating strategy. That's really kind of the main part.

It's really hedging and evaluation and pricing are the same-- right and left hands. We're really talking about the same thing. You cannot really price without hedging. And pricing without hedging is kind of meaningless, in some sense. Pricing represents the price of a hedging strategy.

OK, how do we-- basic credit model-- how do we model default? Well the standard model in finance for default is to define the default events. And we say well, this default event arrives as a discrete event.

And it arrives at the time tau, which is a random time. And we're going to model the tau, the time as a Poisson process, which means that we don't know when it's going to come, but we know something about the probabilities of when it's coming. And the Poisson process has an intensity. The intensity in this case is h.

And basically, the meaning of intensity means the probability of the default time not the arriving by time capital T is e to the minus h times capital T minus little t. Little t means now. Let's say we're saying at time t, we know the default has not arrived. Here is the probability the default will not arrive by some time t later.

So in our model, we're going to make a simple assumption. Let's say constant hazard rate, and we can, since we know the probability of the default time not arriving after a certain time, capital T. That's like a cumulative distribution.

We can also find the probability density the default time happens at some time, capital T, or around some time epsilon, around capital T. It's just the derivative of the cumulative distribution. And corollaries of the probability density of the default at any given time is h, which is essentially the limit of capital T going to little t.

So now in our model, what happens to FX rate? Well FX rate is going to be denoted by s. And FX rate right after default would be equal to FX rate before default times e to the power j. And j, essentially is our kind of percent devaluation, you can think of it. So it's kind of like a percent devaluation. So j can go from minus infinity to infinity.

If j is 0, then that means is there's no devaluation. So you can see the log of ST basically jumps by j. So in a log-normal process, a log of ST is normal, and essentially, it's just a shift of the normal distribution.

OK, so how do we describe this? We define a jump in default Poisson process with intensity h, as on the board. And our FX dynamics-- and I apologize for the small script-- is that our d log of S will have some drift, mu t dt, and then a jump process JdN. So this is slightly different from what you've seen so far. So far you've seen Brownian motions. This is JdN. This is now a jump process.

Now what we want, again, we want still our standard no-arbitrage condition to remain constant. And from before, we had a condition that expected value of S of T has to be S of 0 times e to the rf minus rd times T. So that still has to be the case.

And in our case, we're going to assume that rf and rd are both 0. So in our case, we're going to ask-- basically zero interest rate environment to make, again, the model simple. Then we just want the expected value of ST to be S0.

So how do we achieve that? Well, we need to show, essentially, that this mu, the drift, has to equal to this expression here, h times 1 e to the J. That's known as the compensator term. And you can think-- you can imagine this as a formula.

Like, if I have a Poisson process that has a possibility of jumping up, then in order for that Poisson process to be on average to be equal to the initial value, it has to be kind of trending down most of the time. And then, so that when the possibility of jump is there, the average of the two can be 0. So that's known as a compensator term of the Poisson process.

OK, so we can go through and derive how do we get-- what we want to do is, we're going to check that this form actually does indeed give you that expectation, does satisfy the condition for the expectation. OK, so again, we start with dSt is mu dT. So in our case, it's going to be h times 1 minus eJ times the indicator function of tau bigger than T, plus J dN t. OK, so we're not dSt, sorry. This is d log of ST.

OK, so now what do we want to do? We want to integrate this equation. So essentially, what we're going to do is write integral from 0 to capital T of d of log ST.

We integrate both sides-- integral from 0 to capital T, h 1 minus e to the J, tau is bigger than T, times-- here has dt-- times dt plus integral from 0 to t of J dNt. OK, so then this I just gives me essentially the log of ST over S0. This is just basic calculus.

And then here we have-- we can-- this indicator function just says if tau is bigger than T, it's 1. If tau is less than T, then it's 0. That's basically what it is.

So I know that essentially, this is only 1 when t is less than tau. So my integral goes from 0 to tau now. I can replace this from an integral from 0 to tau. And I can take out the indicator function now. of h, 1 minus e to the J dt.

And then I can say, well, what if tau is bigger than-- there is also a possibility here that tau is-- this is tau is less than capital T. And there's also a possibility that tau is greater than capital T. In which case, if tau is greater than capital T, this integral is there without any indicator functions.

So again, integral from 0 to capital T of h 1 minus e to the J dt times indicator function, tau being greater than capital T. So I kind of divided this, counting both possibilities separately, essentially. And now the second part, integral from 0 to capital t of J dNt. Now, N is-- what was N? Well, N of t is essentially-- it starts out as 0 for t less than tau, and then becomes 1 for t bigger than tau.

So this integral is just-- j is a constant, so it's just j times N of t. And this is capital T here. And by the way, all these derivations are posted on the notes, so you don't necessarily have to worry if you can't-- can I can move this board up? Not really.

So I'm going to do one more line. I'm going to erase this top line. So we get to here, and there's one more step, which is now to actually do the integration.

We're going to have log of ST over S0. Well, two things-- now, if tau is less than T-- so default happened before capital T-- then what is Nt? Nt is going to be 1.

So I can say this equals to h tau times 1 minus e to the J. This is the first-- this integral now-- plus J. So this is if tau is less than T. And then if tau is bigger than T, then this term is 0. This is a term that's for tau bigger than T. This is just a constant, so it just becomes h times capital T, 1 minus e to the J times indicator function of tau bigger than or equal to T.

And we can then exponentiate both sides. And it becomes-- use the magic of the blackboard. You can erase. ST equals S0 times the exponential of this.

So I have-- this is what my exchange rate is going to be, essentially, at time capital T. Now, what was I trying to do? I was trying to do this-- to compute this expectation. With the computed expectation, now I have to integrate over the probability distribution of tau.

Now remember, probability distribution of tau is a Poisson process. So we have essentially-- I'll write it here-- phi of zero t is just h times e to the minus h t. That's kind of the probability density of tau.

So now what I need to do is essentially, the expectation of ST is just the integral from 0 to infinity of S of tau, times phi 0 tau d tau. So here is my S sub T. You can think of this S sub T of tau for a time tau.

So this is for a given time tau, I know what my value of S sub T is. So I can do this integral. And now we're going to do it.

So what is going to be the first term? So exponential of ST-- not exponential, expectation of ST is going to be-- it's going to be integral from 0 to capital T is going to have two terms. First, I'm going to integrate from 0 to capital T. And then I'm going to integrate from capital T to infinity. I'll split this integral into two parts.

And from 0 to capital T, I have essentially-- h times e to the minus h tau. And this is my density function. And then I'm going to plug that in here. So this is for tau being less than T, so it's basically this first term.

I'm going to divide it by 0 here, to make it easy. So first term is going to be e to the h tau times 1 minus e to the J plus J. OK, and so this is d tau. So this is the first part from 0 to T.

And the second part is essentially the integral from capital T to infinity for tau being bigger than capital T. Now that's actually-- this part here does not depend on tau. It's a constant.

So it would be just h capital T 1 minus e to the J times-- what's the probability of tau being bigger than capital T? That's just the cumulative probability distribution we saw before, just e to the minus hT. That's the probability that tau is bigger than T.

OK so it's e to the hT, 1 minus e to the J times e to the minus hT. So I can now simplify this expression somewhat. You can see that, say, this term and this term, this term and this term go away. And also this term and this go away.

So I'm left with the integral from 0 to T of, essentially, h times E to h tau minus h tau e to the J times e to the J. So you can think of this as h times e to the J times e to the minus h tau, e to the J d tau plus e to the minus h capital T times e to the J. So this is-- if I think if h e to the J as this is the constant in front of tau, this is just a standard integral of exponential, so this just becomes, essentially, e to the minus hT, e to the J minus 1 plus e to the minus hT, e to the J.

And these two terms are going to cancel out. And I'm going to have 1. So again, the ratio of e to the ST over S0 just gives you 1.

So all this is just to kind of show you a little bit how you work with jump processes, and take expectations. It's not-- nothing you haven't seen in terms of math. It's just slightly different from Brownian motions. But still the same idea-- you have dN and you have a compensated term. So this here proves that, essentially, my drift guess that I started with, in fact does make my expectation 0.

OK, so what have we done so far? We've defined dynamics for log of S with jump on default, defined probability density. And now we have to derive the dynamics of S, price Euro bonds, hedge ratios, and so on.

OK so log of S dynamics, we-- I apologize again for the small font-- here we have the log S dynamics. Applying Ito's Lemma, there is an equivalent. Ito's Lemma you know from Brownian motion, but there is another one for Poisson processes, as well.

And that is-- Ito's Lemma is like the chain rule. So if you know the process for some log of S, how do you find the process for S itself? Well, in this case, what's going to happen is our dS over S is going to be the same drift-- h times 1, e to the minus J. Tau is less than T dT-- sorry, T less than tau-- plus e to the J minus 1. So J minus 1 is dN, dNt.

So that's really the derivation of the-- that's the final result for S. Now, how do we get to this? Well maybe I should-- I can write Ito's Lemma.

What does it say? Ito's Lemma basically says that if we have dXt is equal mu dt plus

J dN, then-- and you have a function Y of t, which is f of Xt, then dY is df dx mu dT plus f of Xt plus J minus f of Xt dNt. So this is the kind of the term that is kind of an analog of the convexity term in your Brownian motion, Ito's Lemma, but it's now for jump processes.

So this f of Xt plus J and f of Xt-- so what happens, essentially, so you have some function f, and Xt plus J is what happens if a jump happens. And Xt is before the jump, so the effect of the jump on the function. That's what this term is. That's like the convexity term.

I think of as a convexity term. I don't know how it's called. Maybe more mathematical minds here might.

So in our case, if you look at the top, equation our function is just essentially the exponent. And what happens is when the function goes up by J is that the exponent goes e to the J minus e to 0. That's what this term is.

OK, so that's how you write the equation. And now the SDE, solving the SDE generally means write down what S is. So we have S of t.

In our case, it's going to be S of little t. I'm not going to write it. You have it on the board. I think we're going to get late, so hurry up a little bit.

We're going to the next part, which is the pricing exercise. So we have two bonds-- zero coupon, zero recovery bonds. One pays $1. The other pays one Euro. So how are we going to price this?

We have to use our model. We have a model for the FX rate. We have a model for credit. So we price both bonds in dollars. What is the price in dollars for each bond? And the ratio of prices kind of gives you the ratio of the notionals in your hedge portfolio, if you want to hedge one against the other.

So it's a zero coupon bond. So I wrote here the dollar bond price is this. So why do I write it like that?

Well, it's a zero coupon bond. So what a zero coupon bond says is at maturity, it pays 1. So we have something where the payoff at time T is either 1 if tau is bigger than T, or 0 if tau is less than T.

OK, so now what is my price? Well, I know that standard pricing theory tells me that the price of time little t is equal to expectation of a price at time big T. And you can kind of say there is a money market account.

But money market accounts, in our case, is just 1, because interest rates are 0. So that's really just the case. That's just true.

So now the expectation of this-- well, that's just equal to the expectation of an indicator function of tau bigger than T, which just equals to the probability of tau bigger than T. So if that's true, we know what that is. That's just the probability-- that's the cumulative probability function e to the minus hT. That's why the price of the bond in dollars has to be e to the minus hT.

Euro bond price-- same idea, except Euro bond price in dollars is that. So why is the Euro bond price in dollars like that? Well, the Euro bond price in dollars, again, what is the payoff? Same payoff, except the payoff is in Euros, right?

So if I want to do the payoff of my bond in dollars-- so this, I'm going to call this the Euro bond. But the payoff now, if I want to do it in dollars, is not really 1. It's 1 times S of T, and 0 times S of T. That's really my payoff.

So then the expectation here is not just 1, but actually S of T. So now I have something where I have to take the expectation of S of T, essentially, at maturity. My bond price in Euros is equal to the expectation of S of T.

And what is my expectation of S of T? Well, it's e to the minus hT times e to the J. And that's the expectation of S of T, indicator function of tau bigger than T, right?

So not just-- the expectation of S of T is S of 0, but the expectation of S of T times indicator function only in the cases of tau bigger than capital T. Now, that's not 0. That's basically this-- e to the minus hT times e to the J.

OK, so what can we do? Well, we construct a-- what we should do is we construct a portfolio at time equals 0, which is we sell $1 bond, and we buy this much amount here of Euro bonds. And the portfolio value at time equals T equals 0 is 0.

Basically, you can take-- so e to the hT, the first bond, you would get e to the minus hT. And from the second amount would cost you e to the minus hT to buy. That's how I've chosen these scaling factors.

We start a portfolio which costs 0. And I should probably-- I'm going to go back here, and going to write down the notionals, because we lost them. So how many dollar bonds do we have? We have minus 1.

And how many Euro bonds do we have? We have e to the minus hT times 1 minus e to the J. This is how many bonds we have.

OK, so some time, delta T later, what happens to our bond prices? Well, we know what the bond prices are. The only thing that changed was that some time expired.

So now instead of capital T, we have T minus delta T to expiration. So these are the bond prices if we didn't default. Of course, if we defaulted, then the bond prices are 0.

So obviously, if we defaulted, since both bond prices are 0, we started with the portfolios for 0. If default happened, now we have portfolios for 0. So nothing changed, right? So the key part is, OK, now what if default didn't happen? Would we have the same price as well? That's what we want to check.

And if we have the same price, both in the case of default and in the case of no default, then that means we have, essentially, a replicated portfolio-- a hedged portfolio. OK, so what is the value of the bonds if default did not happen? Again, we have these are $1 bonds here, and these are the Euro bonds, and this is my FX rate.

Why did my FX rate move? Well, because default did not happen, so a jump did not happen. But still I had my drift, my compensator drift, so FX drifts in the opposite direction.

OK, so the dollar bonds-- dollar bond that was one bond, minus 1 bond, and the price. So the value of the dollar bond is just minus e to the minus hT minus delta T. What about the Euro bonds?

Well, the Euro bonds-- here is the number of bonds we have. This is divided by a 0, by the way. In our case, that 0 is 1, so it doesn't matter. Price of each bond, again, we take that from-- the price of each bond comes from this formula.

And then the FX rate-- multiply by the FX rate. And then when you actually multiply all these guys out, you end up with, essentially, the value in dollars of you Euro bond equals, again, the value of your dollar bonds. So we started out with a portfolio that was worth 0, and then some time delta T later, it's worth 0 again, both in the case of default and in the case of no default.

So there's no arbitrage. In some sense, not terribly surprising, because we actually derived these prices based on the assumption of no arbitrage. But it's a good check. It kind of tells you, hey, if I actually follow this model to hedge, I'm really going to be hedged. And I'm going to be hedged not just when default occurs, or only if default does not occur, but I'm hedged in both situations-- if default occurs and default does not occur.

And you can't really do that unless you have models that actually are hybrid models-- that allow you to mix and match-- to basically describe both the current event and the FX process. So that's kind of the usefulness. And the hedging strategy you can see-- it's interesting that the hedging strategy-- the hedge ratio depends on the credit riskiness.

So how much bonds we bought depends on J. First it depends on h, the credit riskiness. And it also depends on J, the jump size.

So it really depends. How many bonds you use-- how many Euro bonds you buy to hedge your dollar bonds, it depends on both the probability of default and on the jump size. So that's what I mean by it depends on credit riskiness.

It's also dynamic, in the sense that for a given amount of dollar bonds, the amount of Euro bonds you need to sell is going to vary as FX and time goes forward. As you can see, if you have one day before expiration, the hedge ratio of the two are going to be different than one year before expiration. So you have to be rebalancing your portfolio continuously. Which is not-- again, not unusual. If you're hedging an option, they also have to rebalance.

But it's different from, say, a static replication strategy, where you say, I'm going to buy x amount of Euro bonds, x amount of dollar bonds, and I won't have to ever worry about it. It's not really the case. Here you're saying, well, I buy this ratio of bonds, and if default does not happen, I'm going to have to readjust my ratio.

Because the original ratio took into account the probability of default happening. And if default did not happen, now I have some information-- extra information. And now I have to readjust my ratio to reflect that. So what happens if recovery is bigger than 0? And by the way, how much time do we have-- a quick check?

PROFESSOR: We have till 4 o'clock.

STEFAN ANDREEV: OK. So we have about 12 minutes, 10 minutes. OK, Good. So what happens in case the recovery is bigger than 0?

Well, if recovery is bigger than 0, we can go through this exercise that we did, again, the pricing exercise, and see what happens to our bond prices. So let's do this for dollars and Euro bonds, just to give an example of some of the complexity that can arise when you start making the model more realistic. Because usually bonds do not have zero recovery.

So then we assume that our payoff of the zero coupon, zero recovery bonds was 1 if default doesn't happen, 0 if default happens. Now, it's going to be the payoff of dollar bond at time T is going to be 1 if default did not happen, so if tau is bigger than T, and r if default was less than T. OK, so now when we price our expectation, it's going to be like this.

P of little t would be just expectation at time little t of-- or let's say in this case, I'll call expectation the initial price of 0-- the expectation of P of capital T, which is equal to expectation of essentially 1 of tau bigger than T plus R1 of tau less than T.

Well, what we have here is essentially-- so you can think we have this first guy is going to be e to the-- if tau bigger than T, it's e to the minus hT. And the second guy plus R times the probability of tau being less than T, is 1 minus the probability of tau being bigger than T, so 1 minus e to the minus hT. Which essentially gives you R plus e to the minus hT times 1 minus R. So that's how you derive the dollar bond price.

And for the Euro bond price, you would do the same thing, except now these will be multiplied by the FX rate. And now the FX rate-- the tricky thing about the FX rate is that the FX trade jumps on default. So it's not going to be the same number.

So in this case, PT-- this is for one kind of dollar unit-- it's 1 times S of T and R times S of T. So now we have P little T-- this is for Euros-- the price at time 0 of the Euro bond divided by S0, that equals to expected value times 0 of S of T of tau bigger than T plus R S of T times tau less than T. Well, OK.

The first part, S of T, tau bigger than T, that was like the zero coupon bond price. So that's just essentially, the-- in order to really, I would say guess this well, we have to go back to what was S of T. So if we go back to the equation for S of T, let me write that.

So S of T is S of little t times e to the hT, 1 minus e to the J plus J times 1 tau bigger than T, this is h tau, tau less than T, and this is-- if tau is less than T, and then times e to the hT, 1 minus e to the J, tau bigger than T.

So if default has not occurred, S of T is S of 0-- in this case, S of T is S of 0 times this term. And if default has occurred, then it's S of 0 times this term. So the two terms are the same, except for the J part.

OK, so now when we try to do this expectation, here we're in the situation where tau-- where default has not occurred, so our FX rate is essentially S0 times the second term. So we have expectation of S0 times-- well, and we're kind of dividing by S0, so S0 drops out. e to the hT times 1 minus e to the J. OK, and when tau bigger than T. That's the first expectation.

And the second one, the expectation of-- so we put this R times the expectation of-- now here we have tau is less than T. So we're going to have our S of T is the first part only would be true. Second part would be 1, so that would be the formula-- e to the h tau, 1 minus e to the J plus J times 1, tau less than T.

So this e to the J term that you see here in the Euro price, that comes from this term here. So how do I do this expectation? Well to do this expectation, again, you have to do an integral, essentially, over the interval from 0 to infinity of the probability density.

Since tau here is bigger than T, I'm really integrating from T to infinity. So this here is just a constant. So this first term-- I'll write it here.

So you have P0 over S0, the first term would be e to the hT, 1 minus e to the J. And it's going to be integral from big T to infinity of the partial differential function, so that is just e to the minus hT. So this looks like something we've already done before in the previous calculation.

And then the second term is R times-- now we're integrating from 0 to tau. So this would be integrating from 0 to T, e to the h tau, 1 minus e to the J plus J-- I can do like this, e to the J. Let's put it like that. And times h times e to the minus h tau d tau-- this part being the distribution function, probability distribution function.

So again, we have this guy cancels this. And what we're left with-- first term gives us e to the minus hT e to the J plus R times h times e to the J times tau. This is, again, an exponent function. So we have e to the hT, e to the J minus 1. That's true. Oh, sorry, there's a minus sign here in front of this.

The reason there's a minus sign is we have minus h, e to the J times tau, and so we have to put a minus here in front when we do the integral. So there is a minus here in front.

So this thing just basically reduces to that expression on the board. So that's basically-- so this is how we expand the problem to having no zero recoveries. What you could do for your final paper, if you decide to do a final paper on this topic, is to extend the model one step further, and say, in our model, our FX rates jumped, but did not have any diffusive elements. It was just-- our equation was d log of S was mu dT plus J dNt. That was our SDE for log of S.

So next step would be hey, why don't we just add another term, plus sigma dW? So without the jump, this is just a standard, log-normal process that you know how to do. Now we add jump, essentially.

So you take a log-normal process. You add a jump process to it. And you repeat the same things we were going through so far-- pricing Euro bonds, dollar bonds, and coming up with a replication strategy.

This is, for example, a model that-- we're currently working to implement a model like that at Morgan Stanley. Our model has non-zero interest rates. It has dynamic interest rates. So that makes it a little bit more complex, but overall, it doesn't make it too much more complex.

Having non-zero interest rates just kind of has an extra drift term. It doesn't really change that much the mathematics of it. And the reason why we want to do that is because we want to be able to price, essentially, the contracts which are credit contingent, meaning the payoff depends on whether something has survived or not, whether credit default has occurred or not. And the payoff is in units, anything, like foreign currency.

A typical example would be a credit default swap denominated in Brazilian reals. Or that happens-- a Brazil credit default swap on Brazil denominated in Brazilian reals. Now, common sense is that when Brazil defaults, Brazilian real is not going to cost very much. It's not going to be very valuable, just as we saw on the graph with the Argentinian peso, which totally, devalued it would devalue as well.

Now Brazil is a very big economy, strong country. So right now, people are buying a lot of their bonds. People are investing in it.

Still, it has credit risk. And you can buy-- you can trade the credit risk. You can trade credit default swaps in dollars. And you can also enter into contracts that essentially quanta the credit risk into Brazilian currency itself.

And to be able to really price this, you can do it. We've done it for many years without having a jump model. But then your hedge ratios are not very good. And you cannot really explain the prices you see in the market.

So we are exponentially implementing infrastructure to-- we've already implemented this model or a version of this model, but we're implementing infrastructure to kind of really put it in production. As you can see in this model now, your FX pluses depends on credit. So it actually-- calibration and all these things become a little bit more tricky. Which I don't want to worry about for your final project, but I think it would be a very interesting exercise to take something like that, and basically work out all the steps.

It does get a little bit more complicated, because now you have to-- if you're doing Ito's Lemma, you've got to do it both for diffusive processes and for a jump process, so you're going to have two terms in your Ito's Lemma. But you've seen them both. They're in your class notes. If you're so inclined, you can do it.

And you can-- once you solve the model, then you can kind of check your results. You can actually build a Monte Carlo simulation, or actually run a bunch of paths where you simulate both the default and the diffusive part, and see if your prices arrived at analytically match with your expectations computed by Monte Carlo. This will be a good-- it's always a very good check to see if-- usually, we do this exercise to check if our Monte Carlo simulations is correct, because we know that our math is right. But you can also do it to check the other way around.

OK, so in real life, as we went over-- I mentioned a couple of times during the lecture-- our models are more complicated. We have stochastic interest rates, stochastic hazard rates. So currently, we assume that our hazard rate, h, is a fixed number.

H can be stochastic as well. It can have its own distribution, and typically that's what we use in our models-- stochastic effects. So when I say stochastic, both jump and diffusion processes. And then if you get really fancy, then you can start putting correlations between interest rates, FX and hazard rates.

So in particular, having a jump of FX on default naturally introduces a correlation between credit and FX. When credit occurs, FX devalues. So clearly, there's going to be a correlation.

But there also could be a correlation between the hazard rates themselves and FX. So it's another source of correlation. And these correlations would produce different effects in the market.

So basically, you can, if you have enough data points, you'd be able to say, well, this model seems like it describes the market better than that model. Both of them produce quantum effects, though. And whether we use analytic solutions or Monte Carlo, they're different approaches to price derivatives and compute risk.

It depends, really, on how complex your model is. And for certain markets, you'd rather have a more complex model that is slower and requires Monte Carlo. And in other places, you want to have faster, more tractable models that can price your derivatives analytically.

But maybe your models, they don't have as many features in them. So there's a whole range of models implemented for various markets in Morgan Stanley. It's a very big area of expertise for us.

So I think that's it. I think I ran a little bit over time. I apologize-- five minutes.

PROFESSOR: Thank you very, very much. And we'll thank our speaker first, I guess.

[APPLAUSE]

I think there's probably a question or two that people might have.

AUDIENCE: I was wondering if we could now answer which of the Italian bets was better.

STEFAN ANDREEV: Which what?

AUDIENCE: Which of the bets that we initially were considering on the Italian bonds was better? Could we answer that now? Because we haven't, I think.

STEFAN ANDREEV: Yeah. Yeah, let's go back. Which Italian bonds was better? What was that? OK, so let's try to answer that together.

And we can answer it within our model, right? So in reality, there's all kinds of factors going into the price. So there's supply and demand, liquidity in Euros, liquidity in dollars.

Well, let's say if you're trying to invest in Euros, or trying to invest in dollars, if I invest in dollars, if a default happens, I lose essentially-- let's say the recovery was zero. So I lose all my money in dollars. I thought I had some amount of money in dollars. Default occurs. I lost my dollars.

Same thing in Euros. If I invest in Euros, if default occurs, I lose my Euros.

So how much do I lose in a case of Euros and in the case of dollars? So if I invested Euros, you say, well, if a default happens, my Euros are maybe not as valuable. So Euros are not as valuable, so I lost my Euros, but what I lost was not as much, because already it's also the value of the lot.

Conversely, is we saw, because of the compensator drift-- remember, if you have a jump that makes the currency devalue upon default, the currency will tend to appreciate if default doesn't happen. Because we want the-- the expected value of the currency has to be-- that's determined by interest rates parity, the first thing we talked about-- the interest rate differential.

So that is kind of an ironclad arbitrage condition that we have to follow. So if you want your FX forward to really-- the expected value of your FX to remain fixed by the interest rate differential, and you know that upon default, your currency would devalue, that means if the currency does not devalue, it's going to appreciate. Because if a default does not happen, the currency depreciates relatively speaking.

So in our case, when we're buying bonds, we only get paid if default does not occur. So you would rather, essentially, buy the bonds in the currency that's going to relatively appreciate, essentially. Suppose interest rates were zero in both cases. You would rather buy the bond where FX would appreciate it default does not occur.

Because if it occurs, you get nothing in any case, right? But if it doesn't occur, when you get paid, you want something that would appreciate versus something that would not. So the dollar, for example, let's say the dollar doesn't move versus other currencies when the Euro default happens. So you'd rather get the Euro bonds.

AUDIENCE: If you want to estimate recovery, can you use a bunch-- I mean, not necessarily factors already in the model, but outside factors like macroeconomic factors to predict the expected value of recovery?

STEFAN ANDREEV: Absolutely, yeah. Recovery is something that we cannot really price, necessarily, because usually we have bonds. And the bond price-- you can say we model default, probability of default versus probability of non-default. But now if you introduce a second variable, which is the recovery, now you have essentially both probability of defaults and recovery amount as variables. And you have only price as your data point. And you can have infinitely many solutions.

So typically, what happens is you fix the recovery at something. Now what do we use to fix the recovery? Well, for sovereign countries we use 25% and for corporates we use 40%.

But these numbers-- everybody knows that they're kind of just conventions, really, more than anything. We don't really believe that recovery is really 40% or 25%. It varies a lot by corporation.

And there are studies by credit agencies about how much recoveries-- what are the recoveries for various bonds. And this 25% for sovereign is based on some study like that that went over the last 50 years, looked at the recoveries of sovereigns, of which there are not that many every year. But if you look at 50 years, there's quite a few. And then they made some statements-- some recover higher, some recover lower, but on average, they recover 25%.

If you remember in Greece, what happened in Greece, how much did bondholders in Greece get for their bonds? Now, they didn't really default, technically. Well, they did default technically, but it was a very managed process. But they got definitely less than 25%.

I think they got something on the order of $0.15 on the dollar. So recovery there was, like I say, was less than 25%. Same for this Argentinian default I'm talking about, the 2001-- Argentina is still being sued by creditors trying to get money back from this. And it's a big thing in the news.

AUDIENCE: [INAUDIBLE] if you have a claim from Argentina and they fly over, it can be seized by [INAUDIBLE] funds.

STEFAN ANDREEV: Exactly. They tried to do some settlements. So how much did people recover? Well, it depends who you are.

If you took the original deal, maybe you got $0.20, $0.25, $0.30 on the dollar. Maybe you got $0.20 on the dollar. But now if you hold out-- if you held out, apparently you got a little bit more eventually. So it's a little bit of a fuzzy concept. But it's not something-- you usually make an assumption of what it is.

AUDIENCE: And in a related question, so how would we also estimate the other constants like the hazard rate and the J?

STEFAN ANDREEV: So once you fix the recovery rate, then you can take the bond price. And because bond price directly is e to the minus Ht, you can estimate h from the bond price. So if you observe a bond price in the market, you can say, I'm going to estimate H.

So let's say I'm going to take some benchmark bonds which I know the price of, and

I'm going to estimate H for each of these bond prices. And I'm going to create a curve, which is going to be my hazard curve. And then I take another derivative or bond that I don't know the price of, and I can use the same curve to price it.

So essentially by doing this, what I'm saying is, I'm going to replicate my derivative using these benchmark bonds as much as I can. That's the assumption that I'm making.

AUDIENCE: And how long [INAUDIBLE] if multiple currencies are involved, if we are trying to trade with multiple different currencies, how does the whole model differ?

STEFAN ANDREEV: If multiple currencies are involved, you can-- first you can-- it becomes tricky. You can say each currency can devalue X amount. If default happens, you can have more than one currency being devalued. If you have more than one currency, if you have more than two currencies, like three currencies, there's other identities you have to take care.

You really simulate-- if you have three currencies, there is a triangle identity that, say, dollar-Euros times Euro-yen exchange rate has to equal to dollar-yen exchange rate. That's kind of an arbitrage condition. Just like interest rates, FX forward parity-- even stronger in some sense. And so you can basically, you can write down multiple processes and price stuff.

AUDIENCE: How much do these equations change when you add in bonds that are paying coupons? And how do you factor in duration and all that?

STEFAN ANDREEV: Well, you just-- it's not hard, really. You just, instead of having this, you just write down all the coupon payments, when you pay them. And then you just take an expectation of all the coupon payments. So it's really the same process. You just repeat it for every coupon.

PROFESSOR: Why don't we shut the formal class over now. But if people have questions afterwards, we'll [INAUDIBLE].

STEFAN Yeah, I'm certainly around to answer questions, if anybody wants.

ANDREEV:

PROFESSOR: Thank you very much.

STEFAN ANDREEV: Thank you.