

## 4.8 Another open problem

Feige [Fei05] posed the following remarkable conjecture (see also [Sam66, Sam69, Sam68])

**Conjecture 4.33** *Given  $n$  independent random variables  $X_1, \dots, X_n$  s.t., for all  $i$ ,  $X_i \geq 0$  and  $\mathbb{E}X_i = 1$  we have*

$$\text{Prob} \left( \sum_{i=1}^n X_i \geq n + 1 \right) \leq 1 - e^{-1}$$

Note that, if  $X_i$  are i.i.d. and  $X_i = n + 1$  with probability  $1/(n + 1)$  and  $X_i = 0$  otherwise, then  $\text{Prob}(\sum_{i=1}^n X_i \geq n + 1) = 1 - \left(\frac{n}{n+1}\right)^n \approx 1 - e^{-1}$ .

**Open Problem 4.6** *Prove or disprove Conjecture 4.33.*<sup>21</sup>

## References

- [Fei05] U. Feige. On sums of independent random variables with unbounded variance, and estimating the average degree in a graph. 2005.
- [Sam66] S. M. Samuels. On a chebyshev-type inequality for sums of independent random variables. *Ann. Math. Statist.*, 1966.
- [Sam68] S. M. Samuels. More on a chebyshev-type inequality. 1968.
- [Sam69] S. M. Samuels. The markov inequality for sums of independent random variables. *Ann. Math. Statist.*, 1969.

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