4.8 Another open problem

Feige [Fei05] posed the following remarkable conjecture (see also [Sam66, Sam69, Sam68])

Conjecture 4.33 *Given n independent random variables $X_1, \ldots, X_n$ s.t., for all i, $X_i \geq 0$ and $\mathbb{E}X_i = 1$ we have*

$$\text{Prob}\left(\sum_{i=1}^{n} X_i \geq n+1\right) \leq 1 - e^{-1}$$

Note that, if X_i are i.i.d. and $X_i = n+1$ with probability $1/(n+1)$ and $X_i = 0$ otherwise, then $\text{Prob}\left(\sum_{i=1}^{n} X_i \geq n+1\right) = 1 - \left(\frac{n}{n+1}\right)^n \approx 1 - e^{-1}$.

Open Problem 4.6 *Prove or disprove Conjecture 4.33.*[21]

References

[Fei05] U. Feige. On sums of independent random variables with unbounded variance, and estimating the average degree in a graph. 2005.

[Sam66] S. M. Samuels. On a chebyshev-type inequality for sums of independent random variables. *Ann. Math. Statist.*, 1966.

[Sam68] S. M. Samuels. More on a chebyshev-type inequality. 1968.

[Sam69] S. M. Samuels. The markov inequality for sums of independent random variables. *Ann. Math. Statist.*, 1969.

[21] We thank Francisco Unda and Philippe Rigollet for suggesting this problem.