9/20/04.

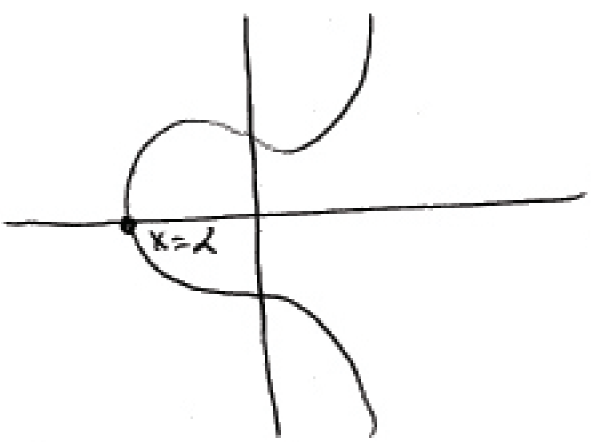
Transformation doesn't map straight lines to straight lines.

$$y^2 = x^3 + ax^2 + bx + c. = f(x)$$

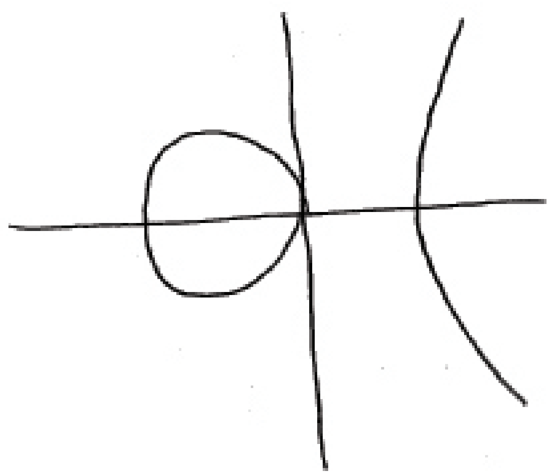
if coefficients a, b, c are rational, the $f(x)$ has at least 1 real root.

$f(x) = (x-\alpha)(x^2+\beta x+\gamma)$

$x = \alpha$ real root of f.



$f(x) = (x-\alpha)(x^2+\beta x+\gamma)$



3 distinct roots

These pictures are valid only if the roots of f are distinct.

Def $F(x,y) = y^2 - (x^3+ax^2+bx+c) = 0$ is nonsingular if $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}$ are never both equal to 0 for (x_0, y_0)

9/20/04.

Let (x_0, y_0) be a singular point on F

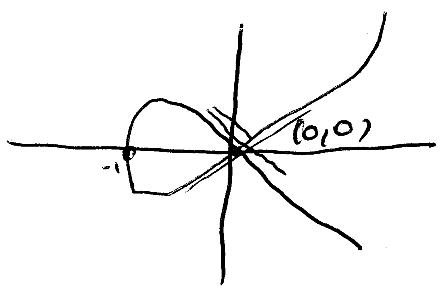
$$\left.\frac{\partial F}{\partial y}\right|_{y_0} = 0 = 2y \qquad \left.\frac{\partial F}{\partial x}\right|_{x_0} = 0 = f'(x)$$

$$y = 0$$

$$y_0^2 = f(x_0) = 0 \qquad x_0 \text{ is a root of both } f(x) \text{ and } f'(x).$$

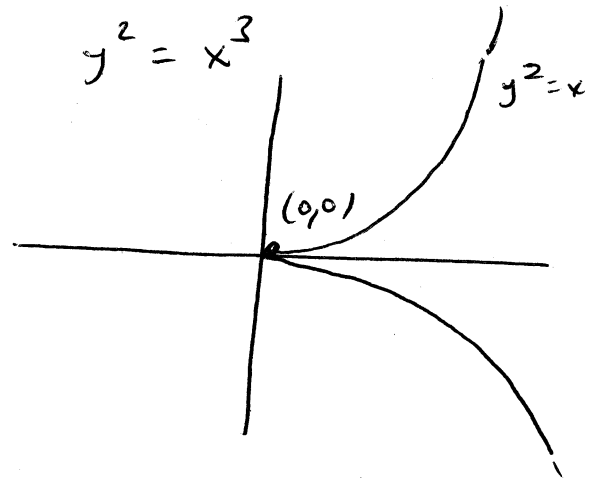
$$f'(x_0) = 0$$

- if f has a double root, then $y^2 = x^2(x+1)$



f has a triple root,

$$y^2 = x^3$$



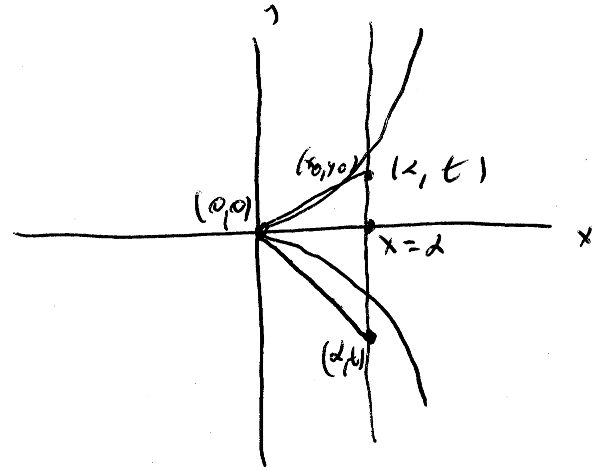
$$y^2 = x^2(x+1)$$

$r = \frac{y}{x}$ $\qquad y = rx$

$r^2 = x + 1$ $\qquad y = r^3 - r$

$x = r^2 - 1$

$$y^2 = x^3$$



$$y = \frac{t}{\alpha} x.$$

$$y_0 = \frac{t}{\alpha} x_0$$

$$y_0^2 = x_0^3$$

$$\frac{t^2}{\alpha^2} x_0^2 = x_0^3$$

$$x_0 = \frac{t^2}{\alpha^2}$$

$$x = \frac{\alpha}{t} y$$

$$y_0^2 = \frac{\alpha^3}{t^3} y_0$$

$$y_0 = \frac{t^3}{\alpha^3}$$