18.336 Numerical Methods of Applied Mathematics -- II
Spring 2009

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Problem Set 3

Out Thu 03/12/09 Due Thu 04/02/09

Problem 6
Consider the 1d Poisson equation

$$\begin{cases} -u_{xx} = f & \text{in }]0,1[\\ u = 0 & \text{on } \{0,1\} \end{cases} \tag{1}$$

with $f(x) = \sin(\phi(x))(\phi_x(x))^2 - \cos(\phi(x))\phi_{xx}(x)$, where $\phi(x) = 9\pi x^2$. Consider a 3-point finite difference approximation on regular grids (grid spacing h) to approximate (1) by linear systems $A_h \cdot u_h = f_h$. Implement a V-cycle multigrid scheme (ν presmoothing and ν postsmoothing steps), which solves systems smaller than 100×100 exactly. Solve the linear system corresponding to $h = 2^{-15}$ by multigrid. For $\nu \in 1, 2, 3$, show the error in the various levels in the V-cycle. How does the final (multigrid) error compare to the approximation error of the finite difference scheme?

Problem 7
On the domain $\Omega =]0,1[^2 \backslash [\frac{1}{4}, \frac{1}{2}]^2$, with the boundaries $\Gamma_D = (\{0,1\} \times [0,1]) \cup ([0,1] \times \{0,1\})$, and $\Gamma_N = (\{\frac{1}{4}, \frac{1}{2}\} \times [\frac{1}{4}, \frac{1}{2}]) \cup ([\frac{1}{4}, \frac{1}{2}] \times \{\frac{1}{4}, \frac{1}{2}\})$, consider the Poisson problem

$$\begin{cases} -\nabla^2 u = 1 & \text{in } \Omega \\ u = f & \text{on } \Gamma_D \\ \frac{\partial u}{\partial n} = \frac{\partial f}{\partial n} & \text{on } \Gamma_N \end{cases} \tag{2}$$

with $f(x,y) = x^3 y - x y^3 - \frac{1}{2}x^2$.

1. Write a finite difference code that approximates (2) for various grid resolutions $h = \Delta x = \Delta y$.
2. Implement a good multigrid solver that solves the arising linear systems. You can restrict to mesh sizes that are powers to 2, for which the domain boundaries fall onto grid edges.
3. Compare the run times (`tic`, `toc`) of your multigrid solver with the run times using the Matlab backslash operator and with conjugate gradients for a comparable accuracy.