

Now that I got it to work, here is the behavior of that horror example of type

$$\begin{bmatrix} 1 & & & & 1 \\ -1 & 1 & & & 1 \\ -1 & -1 & 1 & & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix},$$

devised independently in the early 1960s by Wilkinson and Kahan, illustrating the occasional (but fortunately rare!) inadequacy of partial pivoting, when let loose in an attempt to recover either the vector

$$x = [\pi, e, \pi, e, \pi, e, \dots]^T$$

or else one filled with random numbers drawn from a uniform distribution in the interval $(-1,1)$.

The plot below shows the decimal logarithms of the relative errors found empirically as functions of the size N of the $N \times N$ matrix. The solid dots refer to the π, e example, whereas the open circles give results for the randomly-loaded case. Drawn for comparison is a straight line corresponding to growth like 2^N .

In both cases, the rate of instability almost matches that worst-case line. The leveling out at larger N 's may seem equally dramatic, but then the errors are already of order unity; it gives no cause for celebration.

