18.330 Problem Set 11 Due in class: Fri 30 Apr 04

(31) Figure out carefully the "condition number" $CN \equiv \max\{|\vec{b}||\delta\vec{x}| / |\vec{x}||\delta\vec{b}|\}$ referring to vectors such that $\underline{A}\vec{x} = \vec{b}$ and $\underline{A}(\vec{x}+\delta\vec{x}) = (\vec{b}+\delta\vec{b})$ and to the asymmetric matrix

$$\underline{A} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} .$$

Propose vectors $\vec{x}$, $\vec{b}$, $\delta\vec{x}$ and $\delta\vec{b}$ to illustrate this "worst case" scenario, and also contrast your result with the erroneous answer $CN = \max|\lambda| / \min|\lambda|$ (mis)inspired by the theory for the related symmetric matrix

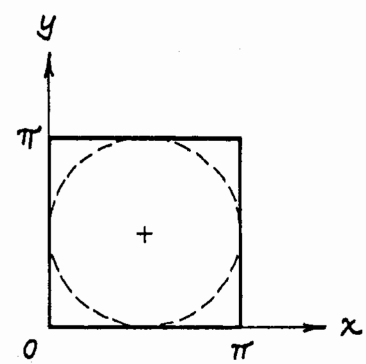
$$\underline{B} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} .$$

(32) Reproduced overleaf is an old **one-hour exam** from the course 18.086. Its three problems should by now look very familiar also to you. So polish them off, please, at your leisure ... here meant **not** as any fresh exam but instead simply as a source of nice exercises!

(33) Determine the sag $u(\pi/2, \pi/2)$ at the center of a uniformly-loaded **square membrane** obeying the Poisson PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$$

and also the sensible requirement that $u = 0$ along all four of its edges of length π .



For that purpose, build yourself a uniform $N \times N$ mesh demanding

$$u_{i,j-1} + u_{i,j+1} + u_{i-1,j} + u_{i+1,j} - 4u_{i,j} = h^2$$

in the interior, and $u_{i,0} = u_{i,N} = u_{0,j} = u_{N,j} = 0$ at the edges.

From this, compute $u_{N/2,N/2}$ once again to at least 6 or 9 decimals with the help of Gauss-Seidel-type **successive over-relaxations** for N = 10, 14, 20, 28, 40, etc. using SOR coefficient ω = approx 1.5 . Follow that by ample Richardson **extrapolations**, since we are seeking this PDE answer really in the limit at N approaches infinity.

PS: Sag $u(\pi/2, \pi/2) = -\pi^2/16$ at the center of a **circular** membrane of diameter π .