18.325 :: Homework 3 :: Fall 2015

In this problem set we will form an image from a fan-beam CT dataset.

Download and load the dataset in MATLAb® with `load siemens.mat`

The array g is a sinogram. It has 513 rows, corresponding to uniformly sampled offsets t, and 360 columns, corresponding to uniform, all-around angular sampling with 1-degree steps in θ. The acquisition is fan-beam: a transformation is needed to recover the parallel-beam geometry. The fan-beam geometry manifests itself in that the angle depends on the offset t in a linear fashion. Instead of being just θ, it is ($1 \leq t \leq 513$ is the row index)

$$\theta + \frac{t - 257}{256}\alpha,$$

with

$$\sin \alpha = \frac{1}{2.87}.$$

Imaging from a parallel-beam sinogram is done by filtered backprojection. Filtering is multiplication by ω in the ω domain dual to the offset t. Backprojection of a sinogram $g(t, \theta)$ is

$$I(x) = \sum_{\theta} g(x \cdot \mathbf{e}_\theta, \theta),$$

where $\mathbf{e}_\theta$ is $(\cos\theta, \sin\theta)^T$. (Why is this the same thing as what we saw in class?) Form the image on a grid which has at least 100 by 100 grid points (preferably 200 by 200). You will need an interpolation routine since $x \cdot \mathbf{e}_\theta$ may not be an integer; piecewise linear interpolation is accurate enough (`interp1` in MATLAb).

In your writeup, show your best image, your code, and write no more than one page to explain your choices.

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