Pak

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Thm (Mantel)

$|V(G)| = 2n$, $|E(G)| \geq n^2+1 \Rightarrow \Delta \subseteq G$

Thm (Turán)

blah

Pf: By induction, $n = r+1$ ✓

$n > r+1$ Let G be a graph on n vertices, $t_r(n)$ edges, w/out K_{r+1}. Claim: G is $T_r(n)$ (this is enough since then it must be subgraph of any graph w/ more edges + no K_{r+1} ✓)

min deg $G \leq$ min deg $T_r(n)$, since $T_r(n)$ is close to average. Let $x \in V(G)$ have min deg. Then $|E(G-x)| \geq |E(T_r(n-1))|$ (since $T_r(n)$ − vx of min deg $= T_r(n-1)$). By induction, $G - x = T_r(n-1) \Rightarrow x$ has deg min in $T_r(n) \Rightarrow$ x has neighbors as in $T_r(n)$ ✓

Thm: $|V(G)| = n$, $|E(G)| \leq nk/2 \Rightarrow \alpha(G) \geq n/(k+1)$

Pf: $\sigma : [n] \to [n]$ random permutation

A_i = event of all neighbors of i having labels $> \sigma(i)$

$P_r(A_i) = 1/(\deg(i)+1)$, since permutations of $\{i\} \cup N(i)$ are equally likely, prob i is smallest is $1/(\deg(i)+1)$

X = set of vertices i s.t. A_i happens

$E(|X|) = \sum_{i=1}^{n} P_r(A_i) = \sum_{i=1}^{n} 1/(\deg(i)+1) \Rightarrow \exists\, \sigma$ s.t. $|X| \geq \sum 1/(\deg\, i+1)$. Also, X is independent, so $\alpha(G) \geq \sum 1/(\deg\, i+1)$ is minimized when $|d_i - d_j| \leq 1$ (from Cauchy-Schwarz or whatever), so

$\alpha(G) \geq n/(k+1)$ b/c $\deg(i) \approx \left(\frac{nk}{2}\right) \cdot 2 \cdot 1/n = k$ ✓