Pak

11/4/05

Jukna p98

Thm (Dilworth, 1950)

P - poset. Then max size of an antichain in P = min size of partition of P into changes

Pf: $\le$: obvious

$\ge$: Use Gallai-Milgram (1960) min size of a path cover $\le$ max size of an independent set in G

Let $V(G) = P$ $E(G) = \{(x,y) \mid x \le y\}$ ✓

Thm

P - poset. Then max size of a chain = min size of partition into antichains

Pf: $\le$ obvious (pigeonhole)

$\ge$ each partition P_i is those elts w/ max descent of length i ✓

Hall Marriage Theorem (1935)

$[n]$, $[k]$, $n \ge k$ $\forall$ $i \in [k]$ $\exists$ $S_i \subseteq [n]$ bipartite graph

$\forall I \subseteq [k]$ $\left|\bigcup_{i \in I} S_i\right| \ge |I| \iff \exists$ matching of $[k]$

Pf: From Dilworth, make poset in obvious way.

\# max antichain = n (by contradiction ✓) ✓

Cor (Dilworth himself)

P poset of size $n \ge pq+1$. Then P contains a chain of size $p+1$ or P contains an antichain of size $q+1$ (or both)

Pf: by contradiction ✓

Thm (Erdős - Szekeres)

$\sigma \in S_n \Rightarrow \sigma$ contains an increasing or a decreasing subsequence longer than $\sqrt{n}$

Pf ~~Write σ as a 0-1 matrix~~, P is a poset on $[n]$ $\quad i \leq_P j \Leftrightarrow i < j \quad \sigma(i) < \sigma(j)$ ✓