+ Tutte

PaK
9/28/05

Chromatic Polynomial! Whoo-hoo!

Def'n $\chi_G(t) = \#$ proper t-colorings of G

"chromatic polynomial"

(defined by David Birkhoff in 1912)

Thm $\chi_G(t)$ is polynomial of deg $n = \#$ vertices in G, highest coeff $= 1$ (i.e. $t^n + \dots$)

Pf: (First some examples. G = path $\chi_G = t(t-1)^{n-1}$

G = tree $\chi_G(t) = t(t-1)^n$)

By induction ✓

$(\chi_G(t) = \chi_{G-e}(t) - \chi_{G/e}(t))$

H. Whitney ~1930

(NBC) Thm $\chi_G(t) = t^n - a_{n-1}t^{n-1} + a_{n-2}t^{n-2} - \dots$

$a_i \in \mathbb{N}$

Pf: By induction again ✓

Thm (Whitney)

$a_{n-i} =$ # of i-subsets of edges w/out broken cycles (∀ linear orderings $\prec$, it doesn't matter)

Def'n: Broken cycle

Let $\prec$ be a linear order on edges of G.

A broken cycle is a cycle $- e$ where e is the smallest elt of the cycle

Pf: By induction, removing largest edge