18.311: Principles of Applied Mathematics Rodolfo Rosales
Lecture 26 Spring 2014

Here we are only interested in real matrices, though the results have easy generalizations to complex matrices.

a) A symmetric matrix has real eigenvalues, and the eigenvectors can be taken to be an orthonormal basis.

b) If {n_j}, 1 <= j <= N, is an orthonormal basis of R^N, then any vector Y can be written in the form
Y = sum_1^N y_j n_j,
where
y_j = <n_j, Y>
< , > = scalar product.

c) Let A be a real-diagonalizable square matrix. Let R_j, 1 <= j <= N, be a set of N linearly independent right (column) eigenvectors of A
A R_j = lamda_j R_j,
where the lambda's are the eigenvalues [they need not be distinct]. Then a set of N linearly left (row) eigenvectors of A
L_j A = lambda_j L_j
can be selected such that
L_n^T R_m = delta_{n, m},
where delta_{n, m} is the Kronecker delta. Then any (column) vector Y can be written in the form
Y = sum_1^N y_j R_j,
where
y_j = L_j^T Y.

This formula generalizes the one in (a-b), which applies for symmetric matrices.