18.100C Lecture 6 Summary

Throughout, (X, d) is an arbitrary metric space. Definition of a compact subset $K \subset X$.

Example 6.1. *A finite set K is always compact.*

Theorem 6.2. *If $K \subset X$ is a compact set and $x \in X$ is a point, then $K \subset B_r(x)$ for some r.*

Theorem 6.3. *If $K \subset X$ is compact, it is also a closed subset.*

Theorem 6.4. *If $K \subset X$ is compact and $E \subset X$ is closed, $K \cap E$ is again compact.*

Theorem 6.5. *If $K_1, K_2 \subset X$ are compact, then so is $K_1 \cup K_2$.*

Theorem 6.6 ("you can run but you can't hide")**.** *If $K \subset X$ is compact and $E \subset K$ is an infinite subset, then E has a limit point (in K).*

Theorem 6.7. *$K \subset X$ is a compact subset of X if and only if K itself as a metric space is compact.*

"K itself as a metric space is compact" means this: given any cover of K by subsets which are open (as subsets of K), there are finitely many of those subsets which already cover K.

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