18.100B : Fall 2010 : Section R2
Homework 3

Due Tuesday, September 28, 1pm

Reading: Tue Sept.21 : relative topology, compact sets, Rudin 2.28-35
Thu Sept.23 : compact sets, Rudin 2.36-44

1 . Let E and F be two compact subsets of the real numbers $\mathbb{R}$ with the standard (Euclidian) metric $d(x, y) = |x - y|$. Show that the Cartesian product

$$E \times F = \{(x, y) \,|\, x \in E \text{ and } \; y \in F\}$$

is a compact subset of $\mathbb{R}^2$ with the metric $d_2(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|_2$.
(Recall that the norm $\| \cdot \|_2$ is defined by $\|(x, y)\|_2 = (x^2 + y^2)^{1/2}$.)

2 . Problem # 12 page 44 in *Rudin*.

3 . Problem # 14 page 44 in *Rudin*.

4 . Problem # 16 page 44 in *Rudin*.

5 . Problem # 30 page 46 in *Rudin*.

6 . (a) Show that, for any $\epsilon > 0$, there is a union of intervals with total length $< \epsilon$ that contains the Cantor set $C = \bigcap_{n \in \mathbb{N}} E_n$ (defined in Rudin 2.44). [*Hint*: $C \subset E_n$, and each of the 2^n intervals in E_n is contained in an open interval of length $(1 + \epsilon)/3^n$].

(b) Show that the Cantor set $C \subset \mathbb{R}$ is compact.

(not for credit) Show that the Cantor set is uncountable – either by fixing the proof of Rudin 2.43, or by using another (e.g. diagonal) argument.

18.100B Analysis I
Fall 2010