18.100B : Fall 2010 : Section R2
Homework 2

Due Tuesday, September 21, 1pm

Reading: Tue Sept.14 : countability, Euclidean spaces, Rudin 1.32-38, 2.1-17
Thu Sept.16 : metric spaces, Rudin 2.15-28

Notes: We will call $N_r(p)$ in 2.18(a) an *open ball*, rather than a neighborhood (which has a different meaning in general topology). The notion of *perfect set* is not important for us.

1 . (a) Explain in your own words the logic of a proof by contradiction.

(b) Show that the set $\mathbb{R}$ of real numbers is uncountable. (You can use Theorem 2.14 for inspiration, but be aware that $0.\overline{9} = 0.9999999999\ldots = 1$.)

2 . (a) Let $I = \{[p, q]\,;\, p \leq q, p, q \in \mathbb{Q}\}$ be the set of intervals in $\mathbb{R}$ with rational endpoints. Show that I is countable.

(b) Let P be the set of all subsets of $\mathbb{N}$. Show that P is uncountable.
[*Hint*: for an alleged bijection $f : \mathbb{N} \to P$, consider the set $\overline{D} = \{n \in \mathbb{N}\,;\, n \notin f(n)\}$.]

3 . Exercise 7, p. 43 of Rudin.

4 . Exercise 8, p. 43 of Rudin.

5 . Exercise 9, p. 43 of Rudin.

18.100B Analysis I
Fall 2010