PROFESSOR STRANG: So I'm hoping you will ask some questions today. So we've had the exam. If you have any questions about grading the two TAs are the people to speak to. Ramis graded Questions one and two, and Peter graded three and four. And they control things. I mean, I can help if there's an emergency, but they would be the right people to speak to. Because they know how they graded the whole test. Before you ask questions, can I just say why this truss -- you remember this six-sided truss -- tied me in a knot and I'm hoping your MATLAB solution will untie that knot. The knot I was in was to find the mechanisms, to find convenient mechanisms because we have, well, I thought we had six bars. It looks like we have six bars. But somebody pointed out after class that that bar six is not very active. It's connecting two supports, can't do anything, and actually that sixth row of the A matrix will be all zeroes. So our matrix, if we include that, there was no use, it doesn't help. So our matrix then really comes from the five bars, so A is then five by, two unknowns, two, two, two, making altogether eight unknowns. So three mechanisms and that's what I'm hoping for.

So when I unwisely drew that picture on the board at the end of the truss lecture, I was only looking for two mechanisms because I was thinking we had six edges, six bars, but really this bar, when I knock out the columns that correspond to that node and that node, there will only be zeroes left in that row. And it's nothing. It correctly tells us that the stretching of that bar is zero but we knew that anyway. OK, I don't know if you've tackled the MATLAB question, and I also don't know whether MATLAB would produce for us-- I mean, you should be able to construct A with a whole lot of square roots of three over two from sine of 60 degrees and maybe 1/2's from sine of 30 degrees. A should look pretty nice, but what the solutions to these mechanisms are, of course solutions to Au=0, and-- Anyway, I'm hoping that we learn from this example. I hadn't intended-- So it's pretty small MATLAB, it's really just the creation of the matrix A. OK, so that was a comment on that, which I added to the homework. Now, I'm ready for questions about the homework, the exam whatever, yes thank you. Oh, good.

AUDIENCE: [INAUDIBLE]

PROFESSOR For trusses, OK.

STRANG:

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah. A^T A immediately, couldn't you? It's not quite so straightforward. And this is like more realistic. I mean, yeah. Ah. I guess, yeah, but remind me what that question was. This was a 2.7 problem one? OK, let me just see what I'm asking for. So, OK, yes. So I only asked about A transpose A, I only asked you for the shape in that question. Oh, and then I asked you for the first row, good for me, yes. I see, the first row so I didn't put you to the agony of writing out the whole thing, but still how do you get the first row, good question.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Oh, I see. Now, well it's not going to be so neat. Let's just think. We could do the first row of A, and the first column of A probably, yeah. So what am I looking for in A transpose A, because it didn't say row one. OK, so row one of A transpose A corresponds to the first-- Oh, so what is row one? Yeah, this is worth thinking about. So this is A transpose A for trusses. And let's just, maybe we could even take this one as an example. If I number the nodes, if that's my first node, A transpose A, you remember that's square. That tells us the edge part, the bar part is built into it. But its size is n by n, its size is five by, no, its size is what, eight by eight, right? It's got to do with the number of-- OK. So its first row, what will its first row be about? It'll be about u H 1, right? The first row of this A transpose A matrix will be about, the first node but more than that, that node has got two things, u H and u V, and we're putting H first. So so I think that the first row of this, let's just draw it. Maybe I'm not saying--

So let that be A transpose A, I see, yeah. A transpose A will be multiplying u H 1, u V 1, and so forth. Right? OK, good. So now this is better. All I'm asking is where do we know there are zeroes in that first row of A transpose A? Where would we know that there are zeroes for this problem rather than-- I won't redraw the one in the book so I'm taking this truss. So if it's this truss, this is the first node, the bars are numbered like this, and of course your MATLAB construction of A, you might multiply out A transpose A. Because in MATLAB that would be so quick. And you could see what it looks like. And where would we expect to see zeroes or not zeroes? Let's see, A transpose A, so I'll have to think. My instinct is that it should only connect two neighbors. So that I would imagine, but I've got-- These are double neighbors, right, there's two people living here. We have to remember. So I see two people living here and two people at home, so I guess I would imagine four non-zeroes for this problem in in the first row.

I would think that that would have somebody on the diagonal. That would be what multiplies u H 1 itself, and then maybe u V 1 is involved, and these two guys. These two and those two would not be involved.

Well, that's only a partial answer, I'm just telling you where the zeroes are, I think. And you're really asking about the non-zeroes, of course. So, yeah. Not so easy. Yeah --

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: -- maybe have to do it. OK, yeah. I think, the first time better to do it, yeah. But that's an excellent question, and maybe we will find a nice, and if somebody does, let me know. A nice way to see that. Somehow I must have felt that was doable when I created that problem. OK. The other thing to say though, that while I'm looking at A transpose A, is remember that it can be created by these element matrices. Yeah, yeah, that's important to say. So there will be five element matrices, right? Because I've got five bars. And the element matrix for this bar, or say for a typical bar, the element matrix for this bar will involve all the u's that are connected to this bar. So two u's there and two u's there, so it'll be four by four, the element matrix here. So this will contribute four non-zeroes to that to row and to three other rows. That four by four guy from this part is, you know what I'm speaking about here? The element matrix which is just-- The element matrix is a cosine, sine, minus cosine and minus sine. Column times row. Cosine, sine, minus cosine, minus sine. This is the element matrix for this bar, those are the cosine and sine of 60 degrees, and those positions would be these two and these two. So there would be actually four zeroes coming after, and after here from those two and those two.

So this bar contributes to A transpose A, sort of stamps into A transpose A. This four by four of non-zeroes plus the rest zeroes, stamps that in up there. This guy has its own element matrix. Now, what's different about the element matrix for this guy? Of course it's got its own c_1, probably, and it had a c_2, this difference constant. But bigger differences than that. First of all, it's a different angle, theta. Here the angle is 120 degrees, and also there's nobody home down there. So it would be, if we were dealing with A_0, free-free stuff, it would be four by four, like all the others. But because these two are fixed, those two displacements are fixed, effectively it will only contribute two by two. A total of four non-zeroes. But those will stamp into A transpose A overlapping this guy. Then the ones from these bars will not touch the first row of A transpose A, so they wouldn't touch the answer to that homework problem. So you see again another way. By hand, I don't know that you necessarily want to use these element matrices, I'm not sure. But it gives an excellent check. So one way to create A transpose A is create a-- Multiply, that's one way. Second way is, create A transpose A element by-- By these five element matrices popped into the right places. I think this is, so this like the computational science part of the course. It's the way you would, I'm really speaking about the way you would write the code, not the final result. So, much of the course has been devoted to understanding A transpose A, and the fact that it's positive definite, or in this case only positive semi-definite because we have an eight by eight matrix whose rank is only going to be five.

So this matrix will have these same mechanisms, and I'm hoping to learn what they are. Well I hope that's a little help with that question. But basically, though, it's non-help because I'm sort of saying you're on your own to actually-- I don't have a superfragilistic way to construct a matrix and then you kind of have to do it. But I'm glad you asked. Yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: More on this problem, OK. The sign convention.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yes. So let's see. So I made a speech about sign conventions, right? Which was that I am too old for them. And the justification for that is the fact that they wash out in A transpose. So if you were to use the wrong sign convention in A, you won't see it in A transpose A. So if I change the whole thing. So sign convention is not really a convention. Our convention is to decide that that movement that way is plus. And movement this way is plus. And here, similarly. That's a plus movement and that's a plus movement. That's our sign convention. If we always do that, then the A matrix is telling us how much this bar stretches. It's the matrix that gives us stretching from these four displacements. And then, these, I think in this picture, if this is the later node and this is the thing, then that angle from the horizontal is the right guy to put there.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Here?

AUDIENCE: [INAUDIBLE]

PROFESSOR Well, I guess I'm, I see, the question is what's the sign convention with this particular picture?

STRANG: Yeah, if this is node one and that's two, then-- Your question is good. It looks, I think, so I think I've got it not right. If this is node one down below, and two up above, then moving two positively would stretch the bar. And I've got minuses there. So I've got the opposite signs. I've got there, the signs that would go with this numbering. If that was node two and this was node one. Yeah, you're right, good. You have to, and that's the only way I do it, is to think will the movement stretch the bar? Then that's got a positive entry in A. Will the movement like that compress the bar, that'll be a negative entry. And our conventions are if it stretches that's positive. e positive means a stretching; e negative means a compression. So it takes a little patience. Or it takes writing the code correctly once and then of course it would do it for all these problems. Yeah, I think the TA in an earlier year actually wrote a code to, a truss code. I don't know what happened to it but I could look.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yes, yeah. That's right, and it will come out consistent. Yeah, if I take horizontal-- if I take the same convention that forces that way are plus, that's a plus f H 1, and it's a plus u H 1, then I'll get A will go to A transpose. Yeah, so if I'm consistent that plus for the u's, for the displacements, and plus for the f's, forces, then the correctly created A will give me the correct A transpose. But A transpose A has the ability to bury some of those conventions. OK. Good. Yes.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Different. More. Oh my God. OK. Yes.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: This one, yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: I wish it would ask about bar three. Yeah, yeah bar three was good. Yeah right. Yeah, I guess here, what's here, well, the angle is, that must be theta, and then this'll be right provided this number is the, is what? Well, I've got to think it out again. I'll just think it out. Maybe, yeah I won't try to, yeah I won't. I won't. So I think you-- But it's really, you learn the point by thinking that through. I think we've got it here. That if that's number one then we like plus signs here because positive displacements will stretch the bar, right. And here this sign will depend on the angle. And it will depend on the number, which numbers, what number that one is and what number that one is. If I reverse the numbers, then I reverse the sign. Yeah, good. Well, that's sort of part of computational engineering isn't it? Like, keep track of the details. But of course this 18.085 course is also about big picture things. And that's what I'm hoping comes through even better.

OK, ready for another question on any topic. Thank you.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yes. Well, no, but let's just try it. Shall we do Problem one? Was it Problem one? In 2.7? OK, let me just draw that truss. And let's just talk about it. So, boy I've put a lot of bars in there. OK, so this is just one bar down to this guy. And this is one up to here. And was that everything? OK, and then I've numbered them. Let me just copy the number here, so that we can talk about that. And I numbered the nodes one, two. One. Joints, I should say really. Four. OK, all right. Let's just start as I always do by, what's the shape of the matrix A? A is what, how many bars have I got? So help me through this now. Six bars, and how many unknown displacements have I got? Eight, OK, good. And have I got any rigid motions here? No. I've got two, these are going to prevent all the rigid motions. So if this matrix, if A has rank six, which we'd probably guess it has, you know that there's an if there. Then two mechanisms, right? Eight minus six gives two mechanisms. OK, and your question is how to find them. You've got one. Alright, which one have you got?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: These guys. So tilt, so that's a mechanism that only involves u H 1 and u H 2 and none of the other u's. OK, that looks good. So that's certainly one. Now we're looking for a second one. What if, anybody suggest one? Let me just give everybody a thought. If you haven't already started on this homework, you're starting now. So I'm looking for another mechanism, and of course you might say well, suppose these bars go this way. Suppose they go to the left, and you know that that's not going to do it right. And why not? Why isn't that an OK second answer? Because it's effectively the same as the first; in fact the mechanism u, how would the u, the one I've drawn and the u for this way, what would we see? Opposite signs. If this describes a u 1 H and a u 2 H positive, the other way they're negative, it would just be the opposite sign, nothing new. OK, so now you've had a look, so tell me. If somebody sees-- To answer your question here, how do you see them? I don't know. Just look harder. Like you know, speaking French. Just say it louder and maybe it'll work. Of course we do have the possibility to create the matrix and look for solutions. But it's more fun to do it this way. And now who's going to suggest another mechanism, another view?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Three and four. Wait a minute, what was that three? Oh, these nodes, OK. These nodes, three and four do what? Oh yes, sorry. Three go up this way. Ah, I see. You're rotating. So this guy goes sort of along this way, you're taking that top square and turning it. OK, so these guys will also turn, right? Yeah, one and two will move on your mechanism. So the idea is take that top thing, and I'm allowed to turn it, I'd say I have to turn it, I have to keep this bar, yeah I cannot stretch it. I can't stretch any bars. So when I do this turn, I can't just keep it in place and turn. No. But I'm going to turn it so that this one goes this way and this one goes this way, and now. Alright you're, you're responsible for telling me about the other two now. What do they do?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Bar four, Will bar four keep its length? Ha. OK, well we're not allowed to use fancy words like four-bar linkage. Because somebody might say what does that mean. OK, does it, do you see that? I think it does. But it's wonderful. If that's the original bar and I bring this up a little, a little, remember, and this down a little, I think that the new bar has the same length. You believe that? No. Some, yeah, but what happened there?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah, this is a good example.

AUDIENCE: [INAUDIBLE]

PROFESSOR Sorry, yeah, certainly bringing that up tended to make the bar shorter, but then moving this

STRANG: down tended to make it a little longer.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Right

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah, I think so. See, that was the key. We went at 90 degrees to five. And by going at 90 degrees, then the stretch in there was only that one minus cosine that was higher order. That's the key. Similarly, here, all these-- So we're avoiding stretching by this trick. So maybe this guy is going down this same way. And this guy is going up that same way. Yeah, because then that bar is just translating, and this bar is translating and these two are doing the same thing, which I believe is not stretching. Is this the answer that maybe somebody already got? And believed in? Yeah. So to answer your original question, it wasn't obvious, was it? But I think that is the right thing. And actually, you could create the A for this problem. Well, it's a bit large, 48 entries, but because four of the bars are horizontal or vertical, you will have many, many, many, zeroes in the A matrix. It would be sort of fun to create the A matrix and then this.

So what is this displacement that I believe in? This u mechanism that we've talked about. Let's see, if I look at one that was positive positive. If I looked at node two, that was positive over but down. So horizontal but down. If I look at node three, that's positive positive. Yeah, one and three is hanging on. And number four is like number two, one and minus one. I think that's the u which should solve Au=0. Yeah. OK. But that's a good example. OK, so that's trusses. What else?

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Thank you.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Oh, well, let's see. That's a good question and now I guess we're, ah. Yeah well, the way I've drawn it at 45 degrees, which is what I wrote here, then I did build in, I did make that a forty-- I did make this 45 or 135 or something. And 45. To get that, if that goes at 45 degrees, then this had better go at 135, right? So that has to be a right angle. If the support was over here then the angle that would have gone off with was changed. Yeah, so very good point, that the numbers I've written down on the picture I drew required these angles to be those nice numbers. I hope you like these trusses a little. I mean, you get some freedom to visualize a little. Good, yes.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Maybe. Let me see. I thought you were going to say, could I have two and three go inwards and one and four go outwards? You don't like that. I would go with that. Is that any good? Well, OK. Yeah, linear algebra spoke there. It's a combination of the other two, yes. If we only got a two-dimensional.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: That one.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: OK.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: OK. But let me-- When you suggested one, I overwrote it. Now, the one you suggested-- When you told me, OK, bring these in, I thought, OK, these have to go out.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: They're not allowed to change lengths, so they can only swing around these pin joints. So the picture of the one that I drew after your question would be, these guys, this guy, let me draw the square, as it was. And then these guys came in a little. These guys went out a little. And we got this.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: No, that's OK. I think this is legitimate, this is a legitimate one. And that bar, every bar now is doing the same kind of thing that this one did. This moved a little, but this compensated and kept the length the same. Your question has led us to another nifty mechanism. It's good, a good one to think about.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yeah. It's movement without stretching. Movement without stretching is the key, yes. And then where does collapse come? So I use the word collapse pretty freely. So the word collapse comes in because since it can move without stretching there's nothing controlling it, it can move-- Well, if we stayed linear, then I could make all those ten and the thing would be even worse, of course. The truth is that if I made those even one or ten I would be out of the linear range. These all should be 0.01's or something. But linear we don't know the difference. OK, I'm glad you've got that suggestion. And now somebody correctly says that this one must be some combination of that and what was the first movement? Oh, the upper two. Which it must be. Amazing, how many-- Put a few bars up there and you got it. Yeah.

And of course, by the way, and don't let me get too far into this discussion, but who is the artist, actually is it Alexander Calder who has, what are they called? You know, they're-- What was it called again?

AUDIENCE: [INAUDIBLE] PROFESSOR STRANG: Well, he has those, that wasn't the word I was thinking of. But he's created these trusses, like with lots of bars and lots of nodes that have some special property. You know, they just, anyway. It touches on art, actually this theory of mechanisms. Yeah it's really quite interesting. But then linear algebra somehow tells you just from these numbers how many mechanisms you're looking for. Which is pretty cool. OK, open for more questions. You probably haven't looked ahead, I mean today's lecture was a-- I hope and I left up there the central topics for the lecture. But I'm, I think, correct that you haven't started on those problems. To know what to ask. Let me ask you a question, which was not in the lecture. Suppose I wanted to use finite differences. It's a little bit like the one on the quiz. So on the quiz we had c, equal ones, and then c=2. And you knew c had to be four by four, so most people correctly got the diagonal as one, one, two, two. Just by sort of common sense. But what would be a finite difference approximation to our equation? So suppose I didn't go to finite elements, but instead I stayed with finite differences, which would be completely fine in 1-D, completely sensible. How would you create a finite difference thing for that? Let me just bring down a blank board and ask.

So you see the equation? Let me write it again here. So I want to replace this equation with a varying-- that has varying c(x). Well, I won't worry about the right side. It's f. What's my K matrix using finite differences for this? We're going to create a finite element K matrix by the weak form, Galerkin trial function, that route. That's coming-- That's very important, that's coming, started today and it'll be completed Friday. But now suppose we were back up to finite differences. What would you take for finite differences there? When the c wasn't there, what did we do? Then we just had second difference, right? How did we get the second difference? It was a first difference of a first difference. I guess I would probably-- This I would probably approximate by $(u_{i+1}-u_i)$ over delta x. That would be that, and so what should I put for c(x)? What would you suggest? c subscript i, so let's mark. Here's i, and here's i+1, and here's i-1. We're going to end up with those three guys involved. So this takes this difference, and then this one will take that one, and then I'll also have a u_i-u_{i-1} over delta x. And somehow that'll be the difference of these differences. But now tell me again, what would be the really cool choice of c? What's your instinct where what value of c to take? As sort of average. I mean you could take it just halfway. I think I would take $c_{i+1/2}$ there. Just as being sort of right. And here I would take $c_{i-1/2}$, halfway along its interval.

And then the second difference takes the difference of it. So now I've got-- I think that's what I would do. That would be my typical, one typical finite difference equation. Would be the difference times its c and I took its c to be symmetric. You could also have taken, if you'd liked, if you wanted to stay at these points, you could do twice as much work and take, let me say, or-- So either $c_{i+1/2}$, or you could average the c_i and the c_{i+1} over two. And both of those would give you that extra accuracy that you pick up from sort of keeping symmetry where you should. So I think, I mean, this would be the quicker one to put, that one would be

OK too.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: This was just for your entertainment not, not-- Yes, go ahead.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Ah, when c was a step yes. When c was a step.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: c_(i+1/2) Probably the step-- Yeah, yeah. Yeah. Let me ask you, what happens in this equation when c jumps? It jumped on the quiz. But I didn't require you to solve the differential equation, only to create the finite difference model, to create the A transpose C A, and most people did it fine. But suppose c jumps. I have some simple right-hand side like one. It's not a jump in the right-hand side I'm interested in. It's a jump in c from one to two. So this is a topic that the book does discuss and maybe I might come back to it in the ordinary lecture. But, while it's in front of us now, the quiz was sort of intended to help you with that. How do I interpret this equation when c has a jump? Well, it's actually, if you look at it right there's no difficulty. That equation is a combination of this equation -dw/dx equals the one. And the equation of c*du/dx equalling the w. I split it out for you in the last part, Problem 4b in the quiz. I really helped you to say OK, take these two separate equations. And now you don't really have a problem. The equation for w, nothing is out of the ordinary there. Jump in c is not even seen. So then you've got the w. Now, you have the c in it with its little jump.

But, so suppose I find w from the first equation. How do I find u? I just divide by the c. And integrate. Yeah. What I'm saying, let me say it, clearly now. If there's a jump in c, that's not a bad thing. Because the point is there's no jump in c*u'. c*du/dx doesn't jump. w doesn't jump. From a jump in c. If c jumps, let it. There's no jump in w, which is the serious unknown. There'll be a jump in du/dx, and that was that e thing that you had on the quiz. There'll be a jump in du/dx to compensate the jump in c, but w is good. That's the message of-- So let me write that down. Even if c jumps, w does not. So my point is when you're looking at w, you're looking at the right quantity. And it deals, this is the sort of general feature that if you look at it right it's not a problem. If you look at it wrong and try to write out that equation, take the second derivative of this is OK, but then if you just try to take the derivative of the jump, you think, my God there's a delta function sitting in my equation, what am I to do. Don't do it. That pair of equations is no trouble. Do you see that point? That a jump in c is really OK. c=0 wouldn't be so good, right? If c went to zero then you have got some problems here, because if c is zero here what's going on? So your material should have some stiffness, some positive stiffness. Zero stiffness would be a problem. Negative stiffness would be really a crazy material, where you add more force and it compresses on you. OK so that was a point I could make also in the lecture.

Other questions. I hope you find these review sessions-- They give a chance to bring out more points. Let me say a little bit about Chapter 2 things that we abandoned without including in the lectures. One very important thing that we didn't touch was the non-linear problems. Where in the end you have to solve a system of non-linear equations. We've only had linear equations, Ku=f, where we almost got to-- And we've certainly found enough to discuss there. But how do you deal with non-linear equations? Well so that's Section 2.6, I guess. And the answer is Newton's method, or some version, there are many versions of Newton's method. So all I'm saying is that's an important topic. How to deal with non-linear equations. And the answer is usually some variation of Newton's method. Depending on how many equations you've got, how bad the non-linearity is, and things like that. It's a case where coding is not so simple. But important. So that's one thing in Chapter 2 that we've passed, because really this is where we want to be today. Other topics, questions of any kind? Yes, thanks.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: This problem, OK.

AUDIENCE: [INAUDIBLE]

PROFESSOR STRANG: Yes

AUDIENCE: [INAUDIBLE] Well, OK. Right, good question. You're right.

AUDIENCE: [INAUDIBLE]

PROFESSOR Yeah, OK. So now suppose that delta of x was in-- delta of x-a, the jump at a, was there. What

STRANG: would change in there? Well, the one would change, right? The one would now be the delta of x-a. So what am I seeing now? What's the jump situation now? Is there a jump in, does w jump now from this delta function? Yes. It did not jump from the jump in c. That did not produce us a jump in w. Why not? Because again, back to the jump in c, you have a bar of copper and you have a bar of steel. And they have different c's. But the force at that point, when it's in equilibrium, the bar's not falling apart, right? The force is the same from above and below; equilibrium holds. It's just that the force involves a c_1 there, and down here the force involves a c_2 multiplying the du/dx, and this stretching. So the stretching rule changes. du/dx, the stretching factor will be different in the two materials. But the force won't be. You really have to keep straight what--

Now, you asked about a delta function, OK. So now if I hang a delta function there, that's like hanging a point load at a point. What happens at that point? Well, we don't have a problem with that. Well, w jumps. The force pulling up is not the same as the force pulling down because there's this extra point load. So the balance there and the balance there, those aren't equal. There's a jump there because of the point load being put there. So that produces a jump in w, but not a jump in u. Not a jump in u, the bar is still holding together. And since time's running out I don't have to ask myself, or ask you, about the worst possibility that occurs to me, which is suppose they happen at the same place. Suppose there's a jump in c and a point load. Do we want to face that, what would happen there? Suppose, a was the place where this jumped. Suppose the point load was here. What's up there? Well, maybe I will able to do that. OK, so I'm putting the point load at the place where the copper and the steel meet. Is w continuous? Is the force the same above and below that joint? No. Because there's these extra terms from the weight. And then once you know the forces, then above the bar you're solving for u with one c, and then below that point with the other c. So w has a plus and a minus and c has a plus and a minus. And you're solving this one, let me make them minus plus. So you're solving this one in one bar, one metal. And c plus and the w plus in the other method. So yeah, you can do it. Shall I just repeat what my overall message is? w is the right thing to look at. That combination c*du/dx is the right thing to look at. And it's what sits there. In those parentheses. OK, good, so that's some review topics. We really have a lot of fun ahead now with finite elements.