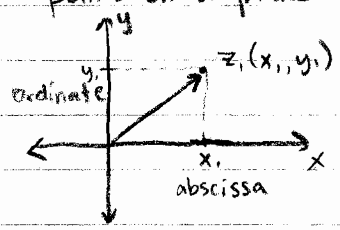
18.075

Fall 2004

Number Systems

1. natural numbers: $m = 0, 1, 2, \ldots$ (positive integers)
   - cannot be used to solve $x + m = n$ where x: unknown; m, n: natural
2. integers: $m = 0, 1, 2, \ldots$ $-1, -2, \ldots$ ← negative integers
   - cannot solve $mx = n$. x may not be an integer.
3. rational numbers: $\frac{m}{n}$; m, n: integers
   - cannot solve $x^2 = m$ (m: positive integer)
4. real numbers; as points of a line
   - cannot solve $x^2 + 1 = 0$
5. complex numbers: $z = \underset{\text{real part}}{x} + i\underset{\text{imaginary part}}{y}$ x, y real $\underset{\text{imaginary unit}}{i}$: $i^2 = -1$

points on a plane



$z_1 = x_1 + iy_1$

Complex numbers can be thought of as vectors.

"length" of vector: $\underset{\substack{\text{absolute value} \\ \text{modulus} \\ \text{magnitude}}}{|z|} = \sqrt{x^2 + y^2} \geq 0$ non-negative

Algebra of Complex Numbers

- addition: $z_1 = x_1 + iy_1$ $z_2 = x_2 + iy_2$

$$z = z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = X + iY = (x_1 + x_2) + i(y_1 + y_2)$$

- multiplication:

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = x_1 x_2 + i(x_1 x_2) + i(y_1 x_2) - y_1 y_2$$
$$= (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

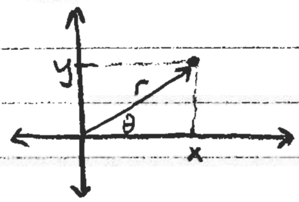
- division:

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{z_1 \bar{z}_2}{|z_2|^2} = \frac{x_2 x_1 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{x_2 y_1 - y_2 x_1}{x_2^2 + y_2^2}$$

Triangle inequality: $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

$z_1 = z_2 \longleftrightarrow x_1 = x_2,\ y_1 = y_2$ but only in Cartesian system

Polar Coordinates (r, θ)



$x = \cos\theta$ $\quad y = \sin\theta$

$z = r(\cos\theta + i\sin\theta)$, $r \geq 0$, $r = |z|$

Specify range of θ: $0 \leq \theta < 2\pi$ (possible)

important! or $-\pi < \theta \leq \pi$