Exam 1 Practice Questions I, 18.05, Spring 2014

Note: This is a set of practice problems for exam 1. The actual exam will be much shorter.

1. There are 3 arrangements of the word DAD, namely DAD, ADD, and DDA. How many arrangements are there of the word PROBABILITY?

2. There are six men and seven women in a ballroom dancing class. If four men and four women are chosen and paired off, how many pairings are possible?

3. Let A and B be two events. Suppose the probability that neither A or B occurs is 2/3. What is the probability that one or both occur?

4. Let C and D be two events with $P(C) = 0.25$, $P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$?

5. Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on this test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?

6. Two dice are rolled.
$A =$ 'sum of two dice equals 3'
$B =$ 'sum of two dice equals 7'
$C =$ 'at least one of the dice shows a 1'

(a) What is $P(A|C)$?

(b) What is $P(B|C)$?

(c) Are A and C independent? What about B and C?

7. A multiple choice exam has 4 choices for each question. A student has studied enough so that the probability they will know the answer to a question is 0.5, the probability that they will be able to eliminate one choice is 0.25, otherwise all 4 choices seem equally plausible. If they know the answer they will get the question right. If not they have to guess from the 3 or 4 choices.

As the teacher you want the test to measure what the student knows. If the student answers a question correctly what's the probability they knew the answer?

8. Suppose that $P(A) = 0.4, P(B) = 0.3$ and $P((A \cup B)^C) = 0.42$. Are A and B independent?

9. Suppose that X takes values between 0 and 1 and has probability density function $2x$. Compute $\text{Var}(X)$ and $\text{Var}(X^2)$.

10. Compute the expectation and variance of a Bernoulli(p) random variable.

11. Suppose 100 people all toss a hat into a box and then proceed to randomly pick out a hat. What is the expected number of people to get their own hat back.

Hint: express the number of people who get their own hat as a sum of random variables whose expected value is easy to compute.

12. Suppose that $X \sim \text{Bin}(n, 0.5)$. Find the probability mass function of $Y = 2X$.

13. Let R be the rate at which customers are served in a queue. Suppose that R is exponential with pdf $f(r) = 2e^{-2r}$ on $[0, \infty)$.

Find the pdf of the waiting time per customer $T = 1/R$.

14. Suppose that the cdf of X is given by:

$$F(a) = \begin{cases} 0 & \text{for } a < 0 \\ \frac{1}{5} & \text{for } 0 \leq a < 2 \\ \frac{2}{5} & \text{for } 2 \leq a < 4 \\ 1 & \text{for } a \geq 4. \end{cases}$$

Determine the pmf of X.

15. Suppose that buses arrive are scheduled to arrive at a bus stop at noon but are always X minutes late, where X is an exponential random variable with probability density function $f_X(x) = \lambda e^{-\lambda x}$. Suppose that you arrive at the bus stop precisely at noon.

(a) Compute the probability that you have to wait for more than five minutes for the bus to arrive.

(b) Suppose that you have already waiting for 10 minutes. Compute the probability that you have to wait an additional five minutes or more.

16. Transforming Normal Distributions
Suppose $Z \sim \text{N}(0,1)$ and $Y = e^Z$.

(a) Find the cdf $F_Y(a)$ and pdf $f_Y(y)$ for Y. (For the CDF, the best you can do is write it in terms of Φ the standard normal cdf.)

(b) We don't have a formula for $\Phi(z)$ so we don't have a formula for quantiles. So we have to write quantiles *in terms* of Φ^{-1}.
(i) Write the 0.33 quantile of Z in terms of Φ^{-1}
(ii) Write the 0.9 quatntile of Y in terms of Φ^{-1}.
(iii) Find the median of Y.

17. Let X and Y be two continuous random variables with joint pdf

$$f(x, y) = cx^2 y(1 + y) \quad \text{for } 0 \leq x \leq 3 \text{ and } 0 \leq y \leq 3,$$

and $f(x, y) = 0$ otherwise.

(a) Find the value of c.

(b) Find the probability $P(1 \leq X \leq 2, 0 \leq Y \leq 1)$.

(c) Determine the joint cdf of X and Y for a and b between 0 and 3.

(d) Find marginal cdf $F_X(a)$ for a between 0 and 1.

(e) Find the marginal pdf $f_X(x)$ directly from $f(x, y)$ and check that it is the derivative of $F_X(x)$.

(f) Are X and Y independent?

18. (**Central Limit Theorem**) Let $X_1, X_2, \ldots, X_{81}$ be i.i.d., each with expected value $\mu = E(X_i) = 5$, and variance $\sigma^2 = \text{Var}(X_i) = 4$. Approximate $P(X_1 + X_2 + \cdots X_{81} > 369)$, using the central limit theorem.

18.05 Introduction to Probability and Statistics
Spring 2014