18.05 Problem Set 2, Spring 2014

Problem 1. (10 pts.) **'Boy or girl' paradox**.
The following pair of questions appeared in a column by Martin Gardner in *Scientific American* in 1959.

(a) Mr. Jones has two children. The older child is a girl. What is the probability that both children are girls?

(b) Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

Be sure to carefully justify your answers.

Problem 2. (10 pts.) **The blue taxi**.
In a city with one hundred taxis, 1 is blue and 99 are green. A witness observes a hit-and-run by a taxi at night and recalls that the taxi was blue, so the police arrest the blue taxi driver who was on duty that night. The driver proclaims his innocence and hires you to defend him in court. You hire a scientist to test the witness' ability to distinguish blue and green taxis under conditions similar to the night of accident. The data suggests that the witness sees blue cars as blue 99% of the time and green cars as blue 2% of the time.

Write a speech for the jury to give them reasonable doubt about your client's guilt. Your speech need not be longer than the statement of this question. Keep in mind that most jurors have not taken this course, so an illustrative table may be easier for them to understand than fancy formulas.

Problem 3. (10 pts.) **Trees of cards.**
There are 8 cards in a hat:

$$\{1\heartsuit, 1\spadesuit, 1\diamondsuit, 1\clubsuit, 2\heartsuit, 2\spadesuit, 2\diamondsuit, 2\clubsuit\}$$

You draw one card at random. If its rank is 1 you draw one more card; if its rank is two you draw two more cards. Let X be the sum of the ranks on the 2 or 3 cards drawn. Find $E(X)$.

Problem 4. (10 pts.) **Dice.**
There are four dice in a drawer: one tetrahedron (4 sides), one hexahedron (i.e., cube, 6-sides), and two octahedra (8 sides). Your friend secretly grabs one of the four dice at random. Let S be the number of sides on the chosen die.

(a) What is the pmf of S?

Now your friend rolls the chosen die and without showing it to you rolls it.. Let R be the result of the roll.

(b) Use Bayes' rule to find $P(S = k \,|\, R = 3)$ for $k = 4, 6, 8$. Which die is most likely if $R = 3$? Terminology: You are computing the pmf of 'S given $R = 3$'.

(c) Which die is most likely if $R = 6$? Hint: You can either repeat the computation in (b), or you can reason based on your result in (b).

(d) Which die is most likely if $R = 7$? No computations are needed!

Problem 5. (10 pts.) **Seating arrangement and relative height**
A total of n people randomly take their seats around a circular table with n chairs. No two people have the same height. What is the expected number of people who are shorter than both of their immediate neighbors?

Problem 6. (10 pts.)
(a) Write down a random sequence of 50 flips (0 and 1).

(b) A run is a sequence of all 1's or 0's. How long is the longest run in your answer to part (a)?

(c) We'll use R to simulate 50 tosses of a fair coin. We'll estimate the average length of the longest run. The code below simulates one trial. You will need to use a 'for loop' to run 10000 trials. In Friday Studio we will go over 'for loops'. We will also post a tutorial on both loops and the rle() function used in the code.

```
# R code to run one trial of 50 flips of a fair coin and find the longest run
nflips = 50
trial = rbinom(nflips, 1, .5) # binomial(1,.5) = bernoulli(.5)
# rle stands for 'run length encoding'.  rle(trial)$lengths is a vector of the
lengths of all the different runs in trial.
maxRun = max(rle(trial)$lengths)
```

(d) A small modification of your code will let you estimate the probability of a run of 8 or more in 50 flips. Do this with 10000 trials and report the result.

18.05 Introduction to Probability and Statistics

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