Part I Problems

Problem 1: Compute the following matrix products:

a) $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix}$

c) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x & u \\ y & v \end{bmatrix}$

Problem 2: Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$. Show that $AB \neq BA$.

Problem 3: Write the following equations as equivalent first-order systems.

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + tx^2 = 0$

b) $y'' - x^2y' + (1 - x^2)y = \sin x$

Problem 4: Solve the system $x' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x$ in two ways:

a) Solve the second equation, substitute for y in the first equation, and solve it.

b) Eliminate y by solving the first equation for y, then substitute into the second equation, getting a second order equation for x. Solve it, and then find y from the first equation. Do your two methods give the same answer?