18.03SC Practice Problems 27

Laplace transform

Rules for the Laplace transform

Definition:	$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt \quad \text{for} \quad \text{Re}\,(s) \gg 0.$
Linearity:	$\mathcal{L}[af(t) + bg(t)] = aF(s) + bG(s).$
$\mathcal{L}^{-1}$:	$F(s)$ essentially determines $f(t)$ for $t > 0.$
s-shift rule:	$\mathcal{L}[e^{rt}f(t)] = F(s-r).$
s-derivative rule:	$\mathcal{L}[tf(t)] = -F'(s).$
t-derivative rule:	$\mathcal{L}[f'(t)] = sF(s) - f(0^-).$

Formulas for the Laplace transform

$$\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[\delta(t-a)] = e^{-as}$$

$$\mathcal{L}[e^{rt}] = \frac{1}{s-r}, \quad \mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[\cos(\omega t)] = \frac{s}{s^2+\omega^2}, \quad \mathcal{L}[\sin(\omega t)] = \frac{\omega}{s^2+\omega^2}$$

1. Use the rules and formulas to find the Laplace transform of $e^{-t}(t^2+1)$.

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