Can Solutions have a Local Maximum?

Quiz: Can Solutions have a Local Maximum?.
Can solutions of autonomous equations have a strict local maximum? (For a function $f(t)$ a strict local maximum at time $t = a$ means $f(a)$ is larger than any nearby values of $f(t)$. Graphically, it is at the top of a hill.)

Choices:

a) Yes.

b) No.

Answer: No.

Suppose $y(t_0) = y_0$ and $\dot{y}(t_0) = 0$ then there is an equilibrium solution $y(t) = y_0$. By the existence and uniqueness theorem this is the only solution with $y(t_0) = y_0$. We have shown that non-constant solutions never have derivative equal to 0, i.e. they don't have any local maxima or minima.

We had to be careful in phrasing the question because constant functions have local maxima, just not strict local maxima. That is, all values of the function are maximum values, but no value of the function is larger than nearby values.

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