Part I Problems

Problem 1: For each of the following autonomous equations $dx/dt = f(x)$, obtain a qualitatitive picture of the solutions as follows:

(i) Draw horizontally the axis of the dependent variable x, indiciating the critical points of the equation; put arrows on the axis indicating the direction of motion between the critical points and label each critical point as stable, unstable, or semi-stable. Indicate where this information comes from by including in the same picture the graph of $f(x)$, drawn with dashed lines.

(ii) Use the information in the first picture to make a second picture showing the tx-plane, with a set of typical solutions to the ODE. The sketch should show the main qualitative features (e.g., the constant solutions, asymptotic behavior of the non-constant solutions).

a) $x' = x^2 + 2x$

b) $x' = -(x-1)^2$

c) $x' = 2x - x^2$

d) $x' = (2-x)^3$

Problem 2: Consider the differential equation $\dot{x} + 2x = 1$.

a) Find the general solution three ways: (i) by separation of variables, (ii) by use of an integrating factor, (iii) by regarding the right hand side as e^{0t} and using the method of optimism (i.e. look for a solution of the form Ae^{0t}) to find a particular solution, and then adding in a transient.

b) This equation is also autonomous. Sketch its phase line and some solutions (including the equilibrium solution). Is the equilibrium stable, unstable, or neither?

c) Use Euler's method with three steps to estimate the value of the solution with initial condition $x(0) = 0$ at $t = 1$.

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