Recitation 5, February 18, 2010

Complex numbers, complex exponentials

1. Mark $z = 1 + \sqrt{3}i$ on the complex plane. What is its polar coordinates? Then mark z^n for $n = 1, 2, 3, 4$. What is each in the form $a + bi$? What is each one in the form $Ae^{i\theta}$? Then mark z^n for $n = 0, -1, -2, -3, -4$.

2. Find a complex number $a+bi$ such that $e^{a+bi} = 1+\sqrt{3}i$. In fact, find all such complex numbers. For definiteness, fix b to be positive but as small as possible. (This is probably the first one you thought of.) What is $e^{n(a+bi)}$ for $n = 1, 2, 3, 4$? (Hint: $e^{n(a+bi)} = (e^{a+bi})^n$.) How about for $n = 0, -1, -2, -3, -4$?

3. Write each of the following functions $f(t)$ in the form $A\cos(\omega t - \phi)$. In each case, begin by drawing a right triangle with sides a and b. (a) $\cos(2t) + \sin(2t)$. (b) $\cos(\pi t) - \sqrt{3}\sin(\pi t)$. (c) $\mathrm{Re}\,\dfrac{e^{it}}{2+2i}$.

4. Find a solution of $\dot{x} + 2x = e^t$ of the form we^t. Do the same for $\dot{z} + 2z = e^{2it}$. In both cases, go on to write down the general solution.

5. Find a solution of $\dot{x} + 2x = \cos(2t)$ by replacing it with a complex valued equation, solving that, and then extracting the real part. Your work also gives you a solution for $\dot{x} + 2x = \sin(2t)$.

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18.03 Differential Equations
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