Problems: Geometric Approach to Line Integrals

1. Let $\mathbf{F}(x,y) = e^x y\,\mathbf{i}$ describe a force field. Show without computation that the work integral along the line segment from $(2,0)$ to $(2,4)$ is 0.

Answer: Since the vector $d\mathbf{r}$ points in the $\mathbf{j}$ direction we have $\mathbf{F} \cdot d\mathbf{r} = 0$. Therefore $\int \mathbf{F} \cdot d\mathbf{r} = 0$.

2. Let C be the curve $g(x,y) = x^3 y + x y^3 = 5$. Find $\int_C \nabla g \cdot d\mathbf{r}$.

Answer: Since C is a level curve for G we know $\nabla g \cdot d\mathbf{r} = 0$. Therefore, $\int_C \nabla g \cdot d\mathbf{r} = 0$.

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