Least squares interpolation

1. Use the method of least squares to fit a line to the three data points

$$(0,0), \quad (1,2), \quad (2,1).$$

Answer: We are looking for the line $y = ax + b$ that best models the data. The deviation of a data point (x_i, y_i) from the model is

$$y_i - (ax_i + b).$$

By best we mean the line that minimizes the sum of the squares of the deviation. That is we want to minimize

$$\begin{aligned} D &= (0 - (a \cdot 0 + b))^2 + (2 - (a \cdot 1 + b))^2 + (1 - (a \cdot 2 + b))^2 \\ &= b^2 + (2 - a - b)^2 + (1 - 2a - b)^2. \end{aligned}$$

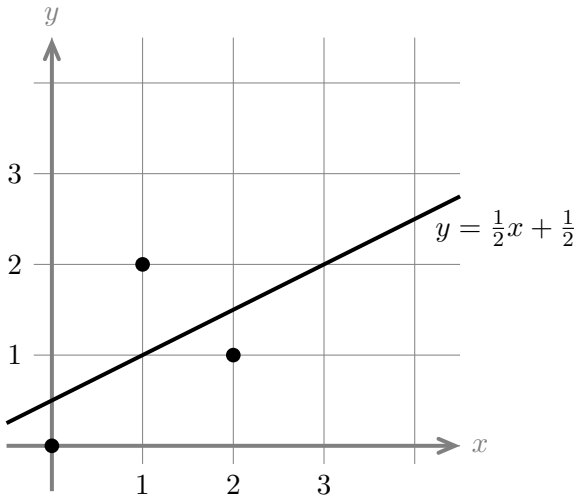
(Remember, the variables whose values are to be found are a and b.) We do not expand out the squares, rather we take the derivatives first. Setting the derivatives equal to 0 gives

$$\begin{aligned} \tfrac{\partial D}{\partial a} &= -2(2 - a - b) - 4(1 - 2a - b) = 0 \;\Rightarrow\; 10a + 6b = 8 \;\Rightarrow\; 5a + 3b = 4 \\ \tfrac{\partial D}{\partial b} &= 2b - 2(2 - a - b) - 2(1 - 2a - b) = 0 \;\Rightarrow\; 6a + 6b = 6 \;\Rightarrow\; 3a + 3b = 3. \end{aligned}$$

This linear system of two equations in two unknowns is easy to solve. We get

$$a = \frac{1}{2}, \qquad b = \frac{1}{2}.$$

Here is a plot of the problem.



18.02SC Multivariable Calculus
Fall 2010