Vector derivatives and arc length

1. Let $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$.

a) Compute, velocity, speed, unit tangent vector and acceleration.

b) Write down the integral for arc length from $t = 1$ to $t = 4$. *(Do not compute the integral.)*

Answer: a) Velocity $= \mathbf{v} = \dfrac{d\mathbf{r}}{dt} = \langle 2t,\, 3t^2 \rangle$.

Speed $= |\mathbf{v}| = \sqrt{4t^2 + 9t^4}$.

Unit tangent vector $= \mathbf{T} = \dfrac{\mathbf{v}}{ds/dt} = \left\langle \dfrac{2t}{\sqrt{4t^2+9t^4}}, \dfrac{3t^2}{\sqrt{4t^2+9t^4}} \right\rangle$.

b) Arc length $= \displaystyle\int_1^4 \frac{ds}{dt}\, dt = \int_1^4 \sqrt{4t^2 + 9t^4}\, dt$.

2. Consider the parametric curve

$$x(t) = 3t + 1, \quad y(t) = 4t + 3.$$

a. Compute, velocity, speed, unit tangent vector and acceleration.

b. Compute the arc length of the trajectory from $t = 0$ to $t = 2$.

Answer: a) Velocity $= \mathbf{v} = \dfrac{d\mathbf{r}}{dt} = \langle 3,\, 4 \rangle$.

Speed $= |\mathbf{v}| = \sqrt{9 + 16} = 5$.

Unit tangent vector $= \mathbf{T} = \dfrac{\mathbf{v}}{ds/dt} = \left\langle \dfrac{3}{5}, \dfrac{4}{5} \right\rangle$.

b) Arc length $= \displaystyle\int_0^2 \frac{ds}{dt}\, dt = \int_0^2 5\, dt = 10$.

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