Parametric curves

1. A disk of radius 2 cm slides at a speed $12\sqrt{2}$ cm/sec in the direction of $\langle 1, 1\rangle$. As it slides it spins counterclockwise at 3 revolutions per second. Measuring time in seconds, at time $t = 0$ the disk's center is at the origin (0,0).

Find parametric equations for the trajectory of the point P on the edge of the disk, which is initially at $(2, 0)$.

Answer: We will parametrize the curve by time t in seconds. To do this we split the motion into translation of the center and rotation about the center and use vectors to do the analysis.

See the figure below. At time t the center has moved to C and the edge point P has rotated $6\pi t$ radians. (3 rev./sec = 6π radians/sec.) Thus

$$\overrightarrow{\mathbf{OC}} = 12\sqrt{2}t \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \langle 12t, 12t\rangle$$

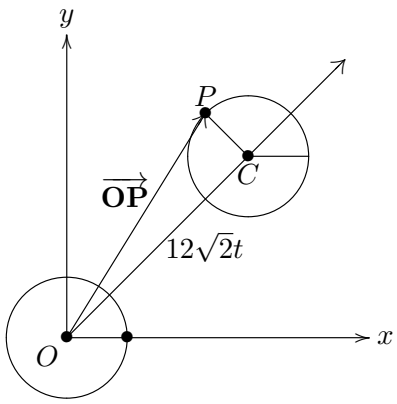
and

$$\overrightarrow{\mathbf{CP}} = \langle 2\cos(6\pi t),\, 2\sin(6\pi t)\rangle\,.$$

Putting these together we get

$$\overrightarrow{\mathbf{OP}} = \overrightarrow{\mathbf{OC}} + \overrightarrow{\mathbf{CP}} = \langle 12t + 2\cos(6\pi t),\, 12t + 2\sin(6\pi t)\rangle$$

$$\Leftrightarrow \qquad x = 12t + 2\cos(6\pi t), \quad y = 12t + 2\sin(6\pi t).$$



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