Meaning of matrix multiplication

In these examples we will explore the effect of matrix multiplication on the xy-plane.

Example 1: The matrix $A = \begin{pmatrix} 4 & 1 \\ 1 & 3 \end{pmatrix}$ transforms the unit square into a parallelogram as follows.

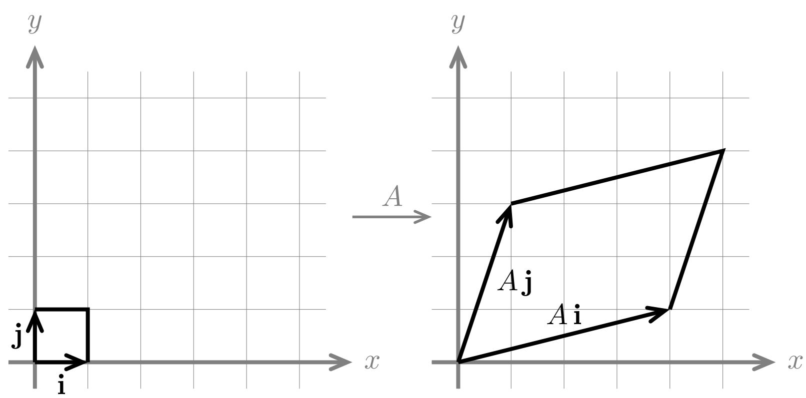
The unit square has sides $\mathbf{i}$ and $\mathbf{j}$. In order multiply a matrix times a vector we write them as column vectors. For example, $\mathbf{i} = \langle 1, 0 \rangle$, $\mathbf{j} = \langle 0, 1 \rangle$ and $\mathbf{v} = \langle a_1, a_2 \rangle$ are written

$$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

The matrix multiplication then becomes

$$A\,\mathbf{i} = A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}; \quad A\,\mathbf{j} = A\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

We think of the all the points in the square as the endpoints of origin vectors. If we multiply A by all of these vectors we get the following picture.



The square is mapped to the parallelogram. We know that the area of the parallelogram is $|A| = 11$. (Think about the 2×2 determinant you would use to compute the area of the parallelogram.)

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