The Indeterminate Form 0^0

We next consider the limit:

$$\lim_{x \to 0^+} x^x.$$

Can we compute this?

There are many different indeterminate forms; x^x is one of the simpler examples. In this case, because x is a moving exponent, we can use a trick to evaluate the limit.

Since we have a moving exponent, we will use base e. We rewrite our original expression as follows:

$$x^x = e^{x \ln x}.$$

Now we can focus our attention on the exponent:

$$\begin{aligned}
\lim_{x \to 0^+} x \ln x &= \lim_{x \to 0^+} \frac{\ln x}{1/x} \\
&= \lim_{x \to 0^+} \frac{1/x}{-1/x^2} \qquad \text{(l'Hop)} \\
&= \lim_{x \to 0^+} -x \\
&= 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\lim_{x \to 0^+} x^x &= \lim_{x \to 0^+} e^{x \ln x} \\
&= e^0 \\
&= 1.
\end{aligned}$$

This was relatively easy to calculate because we have so many powerful tools to work with.