Welcome back to recitation. In this video, I'd like us to do the following problem, which is going to be relating polar and Cartesian coordinates.

So I want you to write each of the following in Cartesian coordinates, and that means our (x, y) coordinates, and then describe the curve. So the first one is r squared equals 4r cosine theta, and the second one is r equals 9 tangent theta secant theta. So again, what I'd like you to do is convert each of these to something in the Cartesian coordinates, in the (x, y) coordinates, and then I want you to describe what the curve actually looks like. So I'll give you a little while to work on it, and then when I come back, I'll show you how I do it.

OK, welcome back. Well, hopefully you were able to get pretty far in describing these two curves in (x, y) coordinates. And I will show you how I attacked these problems. So we'll start with (a).

So for (a)-- I'm going to rewrite the problem up here, so we can just be focused on what's up here. So we had r squared equals 4r cosine theta. Well, we know what r squared is. That's nice in terms of x and y coordinates. That's just x squared plus y squared. So we know that, so we'll replace that. And then we can actually replace all the r's and thetas over here pretty easily, as well. Because we know r cosine theta describes x. So the Cartesian coordinate x is the polar coordinate-- or described in polar coordinates as r cosine theta. So we can just write that as 4x.

And the reason I asked you to describe the curve is because from here, you could say, oh, well I wrote it in the Cartesian coordinates. I wrote it in x, y, and so now I'm done. But the point is that you can actually work on this equation right here and get into a form that you can recognize. That it'll be a recognizable curve. So let's see if we can sort of play around with this, and come up with something that looks familiar.

And what you might think to do, would be, say, you know, subtract off the x squared, or subtract off the y squared. Try and solve for x or solve for y. But that can be a little bit dangerous in this situation, because in fact, y might not be a function of x. So we might run into some trouble there.

But if you'll notice, there's something kind of, a glaring way we should go. And that's because we have this x squared plus y squared together-- this maybe could look something like a circle or an ellipse or something like that, if we could figure out a way to put this part in with the x squared. So this is kind of-- it's a good way to think about what direction to head in this problem.

In particular, it would be a bad idea for this problem for you to subtract x squared and take the square root of both sides. Because you would lose some information about what this curve was. OK? Because when you take the square root, you would have to say, well, do I want the positive square root, or do I want the negative square root? We'd lose a little bit of information. So we do not want to solve for y.

So let's do what I said. Let's try and figure out a way to get this 4x into something to do with this x squared term. So I'm going to subtract 4x and rewrite the equation here. And so you might say, well, Christine, this doesn't really seem that helpful. It's just the same thing moved around. But we're going to use one of our favorite techniques from integration, which is completing the square. So we can actually complete the square on this guy right here, and turn it into a perfect square. We'll have to add an extra term, but once we do that, we'll have a perfect square, an extra term, and a y squared. And we're getting more into the form of something that actually looks like a circle.

So let's see. Completing the square on this, it's going to be x squared minus 4x plus 4. How did I know that? Well, if I want to complete the square on this, I need something that, multiplied by 2, gives me negative 4. That's 2. And then 2 squared is 4. So that's where the 4 comes in. To keep this equal, I'll add 4 to the other side, as well. So if I add 4 to both sides, I haven't changed the equality, and I keep my y squared along for the ride.

So now I have a perfect square. What does this give me? This gives me x minus 2 quantity squared plus 4-- plus 4 squared-- plus y squared. So x minus 2 quantity squared-- that came from these three terms-- plus y squared equals four. And now it's a curve we can describe, clearly.

What curve is this? Well, it's obviously a circle. It's centered at the point 2 comma 0, and it has radius 2. We've talked, or you've seen this in the lecture videos, I believe, what the form for a circle is. x minus a quality squared plus y minus b quantity squared equals r squared. So this is, a is 2, b is 0, and r is 2. so it's a circle of radius 2, centered at 2 comma 0.

So we have a good way to describe what started off in polar coordinates. We can now describe it in (x, y) coordinates.

OK. So now let's move on to (b). And I'm going to rewrite (b) over here as well, so we don't have to worry about it, looking back. r equals 9 tan theta secant theta.

OK. So let's look at this. Now, there's some information buried in here, in terms of (x, y) coordinates. And one thing that should stand out to you is, what is secant theta? Secant theta is 1 over cosine theta. Right? And if we have 1 over cosine theta over here, we can multiply both sides by cosine theta, and we get an r cosine theta over here. So I'm going to write that down. That this actually is in the same-- this is the same as r cosine theta equals 9 tan theta. Right? I mean, you could get mad at me about where this is defined in terms of theta, but I'm not worrying about that in this situation, just right now. We're just trying to figure out how we could write this in x and y.

We know what our cosine theta is. Again, it's x, as it was before. What about tan theta? Tangent theta, remember, if you recall, this tangent theta is opposite over adjacent, right? And in this case, opposite is the y, and adjacent is the x. This is something you saw a picture of, you can see a picture of pretty easily. So this is x is equal to 9 times y over x. Right? Which is x squared is equal 9y.

So this is in fact how you could write this expression that's in r and theta in terms of x and y. And so this, if you look at it, is actually a parabola that goes through the point (0, 0), and is stretched by a factor of 9, or 1/9. Well, I guess you can say, there's a vertical stretch or horizontal stretch, you can pick which one it is. And in one case, it's going to be by 3 or 1/3. I always mix those up. I'd have to check. Or by 9 or 1/9. So essentially, it's going to be a parabola with some stretching on it. Now, the problem is that you might say, well, it's not really all of that. Because secant theta is not going to be defined for all theta the way cosine is. So you do potentially run into some problems. You might have to worry about what part of the domain makes sense for theta, so that this is well-defined. And so that this is well-defined, what part of the curve is carved out. That's a little more technical than I want to go in this video. But some of you might look at it and say, oh, she's missing something. Yeah, you caught something that I'm intentionally ignoring.

So the main point of this was just so that you could see how you can take these functions of r and theta and turn them into functions of x and y, and then figure out kind of what the curves might look like.

So I'm going to stop there. Hopefully this was a good exercise to get you understanding how these different coordinates relate to one another. And yeah, that's where we'll leave it.