Simple Examples in Polar Coordinates

We've just learned about the polar coordinate system, which is very useful in multivariable calculus and in physics. Here are some examples to help you get used to it.

Example: $(x, y) = (1, -1)$

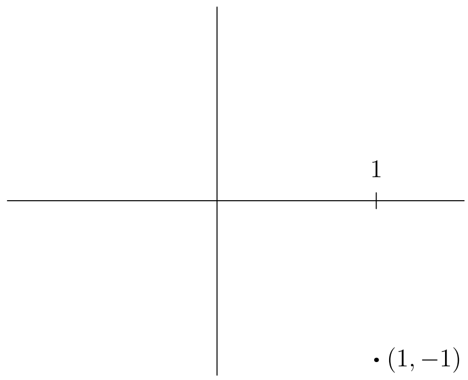

Figure 1: Point at $(1, 1)$ in rectangular coordinates.

How do you describe this point in polar coordinates? There's more than one right answer.

1. $r = \sqrt{2}, \qquad \theta = \frac{7\pi}{4}$
2. $r = \sqrt{2}, \qquad \theta = -\frac{\pi}{4}$
3. $r = -\sqrt{2}, \qquad \theta = \frac{3\pi}{4}$

Some of these answers are easier to make sense of than others, but they are all "legal" correct answers. This ambiguity is something you'll have to adapt to when working with polar coordinates.

Question: Don't the radii have to be positive because they represent a *distance* from the origin?

Answer: When we said r was the distance to the origin in polar coordinates, that was a lie. The only unambiguous description of polar coordinates is:

$$x = r\cos\theta, \qquad y = r\sin\theta.$$

All the others are flawed in some way, but still useful.

It's useful to think of r as a distance, but it's not always accurate to think this way.

Example: $r = a$

In polar coordinates, $r = a$ describes a circle of radius a centered at the origin.

Example: $\theta = c$

The equation $\theta = c$ describes a ray in polar coordinates.

Warning: This implicitly assumes that $0 \le r < \infty$. If we instead assume $-\infty < r < \infty$ we'd get a line, not a ray.

Typical Conventions in Polar Coordinates

- $0 \le r < \infty$
- $-\pi < \theta \le \pi$ **or** $0 \le \theta < 2\pi$.

These are typical, but not universal; different assumptions and ranges are used in different contexts.