CHRISTINE BREINER: Welcome back to recitation.

I want us to work a little more on finding anti-derivatives. In particular, in this video we want to find an anti-derivative of a trigonometric function, a power of the tangent function. So I would like you to find an anti-derivative of tangent theta quantity to the fourth. And the hint I will give you is that you're going to need some fairly familiar, hopefully, by now, trigonometric identities to get this to work. And then you will need some other strategies that you've also been developing.

So I'll give you a while to work on it and then I'll be back and I'll show you how I did it.

OK. Welcome back.

We want to, again, we want to find an anti-derivative for tangent theta quantity to the fourth. And I mentioned that what we're going to need is a particular trigonometric-- well, I didn't say particular, sorry-- but we will need some trigonometric identities to make this work. And the one in particular I'll be exploiting is a certain one, which I'll write down here, which is, 1 plus tangent squared theta is equal to secant squared theta. We've seen that, I think, a fair amount now, but just to remind ourselves, it is important, and this is the one we're going to use.

So let me show you how this works, how this identity will be very useful in here. And the idea is that we can break up this tangent to the fourth theta into two, the product of two tangent squared thetas. So I can rewrite this integral above as the integral of tangent squared theta times another tangent squared theta. But instead of that, I'm going to use this identity. So I'm going to write it as secant squared theta minus 1 d theta.

So just let me make sure everybody follows what I did. I had tangent to the fourth theta as my initial integral, so then I wrote it as tangent squared times tangent squared. And this is actually equal to tangent squared. You notice I just subtracted 1 from both sides of the starred identity. So that's my other tangent squared. So these two integrals are actually equal. So I haven't changed anything fundamentally at all in the problem.

All right. Now let's look at what we get here. We get, if I distribute this, I get integral of tan squared theta secant squared theta d theta minus the integral of tan squared theta d theta.

Now, you should start to see that maybe even powers of tangent theta are nice to deal with. Because this kind of stuff is going to happen, this is going to happen every time with, you know, n minus 2, the power n minus 2, here, for any n I have up here. And the reason the even is-- actually, I guess the even doesn't even really matter. I could just have any n and this would be n minus 2 down here.

This is an easy integral to deal with. Why is that? Because what's the derivative of the tangent function? It's secant squared. Right? So this is actually a straight up u-substitution, or substitution type problem. So this, I can finish and I will later, but this will be substitution. So I'll finish that in a little bit.

But what about this? Now I have tan squared theta d theta. That's, you know, we don't have any, we don't have a secant here. We don't have secant squared here, which would make it obviously nice-- that's what we had here. So we need to do something else with this. Well, what I'm going to do is, I'm again going to use the trigonometric identity. I'm going to replace this tangent squared theta by secant squared theta minus 1.

Let's think about, why is that good? Well, that's good because what I end up with is, if I have secant squared theta minus 1-- is what this will equal-- secant squared theta is easy to integrate. Because it's the derivative of a trig function we know-- it's the derivative of tangent. And 1, I think, is pretty easy to integrate, too. So we have two functions we can integrate very easily.

So I'm going to bring this back up on the next line, I'm going to do the replacement here, and then we'll finish the problem.

So let me write this down. OK. Now I'm going to do my replacement and minus the quantity the integral secant squared theta minus 1 d theta. Let's make sure I didn't make any mistakes. So I had tan squared theta secant squared theta d theta. That looks good. And then I'm subtracting tan squared theta, the integral of tan squared theta d theta. And that's that. So I'm OK.

So this one, again, I mentioned that this is going to be a substitution. If you need to write it out explicitly, this is u equals tan theta, so du is equal to secant squared theta d theta. So this is the integral of u squared. Right? If I substitute in I get u squared du. So it's the integral of u squared, which is u cubed over 3.

So that first part is going to be tan cubed theta over 3. That's my first term. That's a straight up substitution pretty similar to what you've seen.

Now I have two things to integrate. I have to integrate secant squared theta, and I have to integrate the 1. Well, derivative of tangent is secant squared. So the integral of secant squared theta is just tangent theta. So I have to subtract, so there's a minus sign, a tangent theta. And then I have a minus, minus, so when I integrate 1 d theta, I'm going to get a plus theta. And then, obviously, because it's a family of possible solutions, I can add a constant there, plus c.

So again, where did these come from? This first one was a u-substitution on the first integral. And then over here I have another integral with two terms inside. The first one is just the, I just need to integrate secant squared. I get tangent theta. The second one, I just need to integrate the 1, and so I have a negative, negative. That makes it a positive theta. And then I have to add my constant.

So let's come back and just remind ourselves where we started. We started with this trig function that was a power of tangent. And what we ultimately did is we took two of the powers of tangent, we made a substitution with the appropriate trigonometric identity to make this an easier problem to solve. And actually-- yeah-- if you take two of the powers of tangent away and replace them by this, then you're always going to end up with something of this form, tangent to the power 2 less times secant squared theta d theta, which you can always handle by a u-substitution.

And you're going to end up with an integral-- now here's where it gets a little tough-- here, you wouldn't have tangent squared. I think I might have said that incorrectly earlier. If this was any power, here, you wouldn't have tangent squared. You would have had whatever-- if this was power n, this would be power n minus 2, so this would be power n minus 2. Right? So if this was power 8, when we do the substitution, this'll be power 6, so this would be power 6, so this would be power 6.

So you'd have to do the process again. This should remind you of the reduction formulas you've seen. So it's good of it's even, because if this is power 6, you do the problem again and you end up with, the second term has a power 4. You do the problem again, the second term has a power 2, and, oh, we know how to deal with those. So we like it when it's an even power.

So that's kind of how these even powers of tangent, you can take, you can find anti-derivatives of the even powers of tangent by this strategy that winds up, you could actually get a reduction formula out of this.

But I think that's where I should stop with this problem, and I hope you enjoyed it.