Integral of Secant

$$\int \sec x \, dx = ?$$

This calculation is not as straightforward as the one for the tangent function. What we need to do is add together the formulas for the derivatives of the secant and tangent functions.

$$\begin{aligned} \frac{d}{dx}(\sec x + \tan x) &= \sec^2 x + \sec x \tan x \\ &= (\sec x)(\sec x + \tan x) \end{aligned}$$

Notice that $\sec x + \tan x$ appears on both sides of the equation here. If we let $u = \sec x + \tan x$ and substitute, our equation becomes:

$$u' = u \cdot \sec x.$$

Which tells us that:

$$\sec x = \frac{u'}{u}.$$

We've seen this before; this is called the *logarithmic derivative*:

$$\frac{u'}{u} = \ln(u).$$

Putting this all together in order, we get:

$$\begin{aligned} \sec x &= \frac{u'}{u} \quad (u = \sec x + \tan x) \\ &= \frac{d}{dx} \ln u \\ \sec x &= \frac{d}{dx} \ln(\sec x + \tan x). \end{aligned}$$

Integrating both sides, we get:

$$\int \sec x \, dx = \ln(\sec x + \tan x) + c.$$

By taking the derivative of exactly the right function and looking at the results in the right way we got the formula we needed. You won't be expected to do this yourself in this class.

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