Summary of Examples

Now let's look at our results, comparing the function $f(x)$ to the area $\int_0^b f(x)\,dx$ under the graph of f between 0 and b.

$f(x)$	$\int_0^b f(x)\,dx$
x^2	$b^3/3$
$x = x^1$	$b^2/2$
$1 = x^0$	$b = b^1/1$

It looks as if a good guess for $\int_0^b x^3\,dx$ should be $b^4/4$, and in fact this guess is correct.

Historically, Archimedes figured out the area under a parabola in the third century B.C. He used a much more complicated method than is described here, and his method was so brilliant that it may have set back mathematics by 2,000 years. It was so difficult that people couldn't see this pattern, and couldn't see that these kinds of calculations can be easy. They couldn't get to the cubic, and even when they did they were struggling with everything else. It wasn't until calculus fit everything together that people were able to make serious progress on calculating these areas.

We now have easy methods for computing these volumes; we will not have to labor to build pyramids to calculate all of these quantities. We will be able to do it so easily that it will happen as fast as you differentiate.